Blind deconvolution of multichannel recordings by linearized inversion in the spectral domain

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ABSTRACT
Blind deconvolution aims at recovering both the source wavelet and the Green’s function (e.g. reflectivity series) from a recorded seismic trace. A multitude of algorithms exist which tackle this ill-posed problem by different approaches. Making assumptions on the phase spectra of the source wavelet and/or the statistical distribution of the reflectivity series are useful for single traces. The nature of closely spaced multichannel recordings enables a better estimation of a common source wavelet and thus increases the confidence of the results. This approach has been exploited in the past, although different types of assumptions are used for a variety of algorithms. I introduce a new method for simultaneous reconstruction of arbitrary source wavelets and local vertical reflectivity series from teleseismic earthquakes. It is considered that closely spaced receivers record vertically incident earthquake body waves and their surface related multiples which comprise the unknown reflectivity series. By assuming a common source wavelet for all receivers, the observation of several events results in a set of convolution equations relating the unknown source wavelets and unknown reflectivity series to the observed seismic trace. The overdetermined system of equations is linearized and solved by conventional inversion algorithms in the spectral domain. Synthetic tests indicate a better performance of the introduced method than conventional deconvolution in the presence of white noise, which is attributed to the constraint of a common model for all observations. Application to field data from a local deployment allows imaging a basement reflector from teleseismic body waves, although the data are contaminated with strong coherent noise. From a practical point of view, the presented method is potentially well suited for local and regional large-scale imaging from multichannel passive seismic data.

Key words: deconvolution, reflectivity structure, inversion

1 INTRODUCTION
Blind deconvolution is a term coined in signal processing theory and aims at reconstructing both the unknown source wavelet \( w(t) \) and the unknown transfer function \( r(t) \) from the observed convolution \( z(t) = r(t) \ast w(t) \). It is obvious that assumptions on either \( w(t) \), \( r(t) \), or both, have to be made to derive a solution. Conventional deconvolution might be regarded as a special case of blind deconvolution since an estimate of \( w(t) \) can be obtained from the autocorrelation of \( z(t) \) in case \( r(t) \) is white. Since the autocorrelation provides the amplitude spectra only, premises on the phase spectra of \( w(t) \) (e.g. minimum-phase) have to be made. Wiggins (1978) introduced the minimum entropy deconvolution method based on kurtosis maximization of the transfer function. Minimum entropy deconvolution does not rely on the minimum-phase assumption, but requires a broad-band source wavelet. To circumvent the broad-band pre-condition, even more advanced statistical approaches have emerged in the field of electrical engineering (Cadzow and Li, 1995) as well as in seismic exploration (van der Baan and Pham, 2008). In earthquake seismology, multichannel recordings are often utilized to obtain an estimate of the source wavelet by simply averaging the recordings. If the source wavelet of an earthquake is stationary with respect to the receiver array aperture, the average wavelet of all receivers can be deconvolved from the individual recordings to obtain \( r(t) \). This method is used to image lateral sub-surface
variations on basin scale (e.g. Yang et al. (2012)) and on crustal scale (e.g. Tseng and Chen (2006)). Simultaneous least-squares deconvolution of several seismic events (Gurrola et al., 1995) is an alternative approach. Based on the theory of homomorphic systems, multichannel recordings can be transformed into the log-spectral domain where averaging leads to an improved estimate of the source wavelet (Otis and Smith, 1977). This approach has been used in exploration seismology (Tria et al., 2007) as well as in earthquake seismology (Bostock and Sacchi, 1997). The latter study makes uses of multisource and multichannel recordings to sharpen later teleseismic arrivals (e.g., reflections from the mantle-core boundary). Bostock (2004) revisits these concepts in a more elaborate framework which includes the evaluation of three-component recordings to get more insight into the crustal structure on the receiver side. From a technical point of view, many of the aforementioned studies focus on the reconstruction of the source wavelet in the spectral (or log-spectral) domain which in turn is deconvolved from the data. In contrast, Kaaresen and Taut (1995) introduce a method for multichannel data and sparse reflectivity which estimates $w(t)$ and $r(t)$ by a quasi-simultaneous, iterative scheme in the time domain.

Although by far not exhaustive, the list of cited studies illustrates that blind deconvolution finds its application in both exploration and earthquake seismology. Bridging the gap between these two fields is further facilitated by the industry’s growing attention for passive seismology and broad-band data. In particular the interest in the low frequency spectrum of seismic data is mainly driven by the need for robust initial velocity models for full waveform inversion (Sircue and Pratt, 2004; Denes et al., 2009). Passive seismology is well suited to provide those low frequency data due to the instrument specifications and the wide range of possible sources (e.g. cultural noise, regional and global seismicity). Passive seismic methods might also be employed to get a large-scale image of unexplored terrain without utilizing active sources. I introduce a new blind deconvolution method for imaging local vertical reflectivity series from teleseismic events which were recorded on a passive array in south-western Wyoming. The data were acquired in academia-industry cooperation to investigate the feasibility of passive seismology for large-scale subsurface characterization. Therefore, the presented study also aims to contribute to the aforementioned topics.

2 METHOD

A set of $N$ seismic events is recorded on an array comprising $M$ receivers. Those receivers are closely spaced, such that the source wavelet $w_n(t)$ of each event is invariant with respect to the receiver location. It is further assumed that the incoming wave is a vertically incident plain wave. While the latter constraint is satisfied by teleseismic events with epicentral distances greater than 30 degrees in case of small array aperture (<100 km), the validity of the wavelet invariance assumption mainly depends on two factors. These are the size of the Fresnel zone, and the size and magnitude of the impedance discontinuities along the ray path between the source location and the maximum imaging depth at the receiver side. If the spatial extent of these velocity discontinuities is large in comparison to the array aperture, the source wavelets at each individual station are affected by the same transmission coefficients and are identical at each receiver. The local vertical reflectivity series at each receiver station is denoted by $r_m(t)$, and the duration of $r_m(t)$ corresponds to the aforementioned maximum possible imaging depth. The observed seismic trace $z_m(t)$ is associated with the $n$-th event recorded at station $m$. It is modeled by the superposition of the incoming wave $w_n(t)$ and its convolution with $r_m(t)$ after the reflection at the free surface:

$$z_m(t) = s(w_n(t), r_m(t)) + noise(t)$$  \(= w_n(t) - r_m(t) \ast w_n(t) + noise(t) \)  \(1\)

In equation (1), the star sign denotes convolution. Taking the superposition of the wavelet and the convolution term instead of convolution only is of crucial importance when the duration of the wavelet is equal or larger than the zero-offset two-way travel times of the expected reflections (or in other words, when the weaker reflectivity responses are buried beneath the strong source wavelet; Figure 1). The negative sign of the convolution term in equation (1) results from the reflection coefficient at the free surface (-1) and the desire for using the convention nomenclature for reflectivity, where positive impedance contrasts are described by positive reflection coefficients. While all statements so far have been made with P-waves in mind, they are also valid for S-waves where the free surface reflection coefficient is +1, and positive impedance contrasts are characterized by negative reflection coefficients (Aki and Richards, 2009). It should be noted that $w_n(t)$ does not represent the exact source-time function of the earthquake, because it is imprinted by all transmission coefficients between the source and the receiver. To fulfill the aforementioned invariance assumption for the source wavelet, it is required that the product of the transmission coefficients between the imaging depth and the receiver surface is constant over all receivers. This is guaranteed if the targeted discontinuities extend throughout the imaging area, and if the impedance contrast constant along this discontinuities is constant. Further, equation (1) is only valid if both the surface and the discontinuities are sub-horizontal.

In the spectral domain, equation (1) becomes the complex-valued equation

$$Z_{nm}(f) = S(W_n(f), R_m(f)) + Noise(f)$$  \(= W_n(f) - R_m(f) \cdot W_n(f) + Noise(f) \)  \(2a\)
In the following, the frequency \((f)\) is dropped from the notation for simplicity. The function \(S(W_n, R_m)\) in (2a) is explicitly rewritten in terms of real and imaginary parts:

\[
S^R(W_n, R_m) = W_n^R - W_n^R \cdot R_m^R + W_m^I \cdot R_m^I
\]

\[
S^I(W_n, R_m) = W_n^I - W_n^R \cdot R_m^I - W_m^I \cdot R_m^R
\]

where the superscripts \((R, I)\) indicate real and imaginary parts, respectively. If the noise is neglected, 2 \(x\) N \(x\) M observations \((Z_{nm}^R, Z_{nm}^I)\) and 2 \(x\) (N + M) unknowns \((W_n^R, W_n^I, R_m^R, R_m^I)\) are derived for each frequency component \((f)\). If either N or M is larger than 2, these equations become (over-)determined and are solved by linearization with respect to the unknowns (e.g., Rawlinson and Sambridge (1998)). Starting from an initial model \((W_{nm0}^R, W_{nm0}^I, R_{nm0}^R, R_{nm0}^I)\), the linearization is described by

\[
\Delta S_{nm}^R = Z_{nm}^R - S^R(W_{nm0}^R, W_{nm0}^I, R_{nm0}^R, R_{nm0}^I)
\]

\[
\Delta S_{nm}^I = Z_{nm}^I - S^I(W_{nm0}^R, W_{nm0}^I, R_{nm0}^R, R_{nm0}^I)
\]

\[
\frac{\partial S^R}{\partial W_n^R} \cdot \Delta W_n^R + \frac{\partial S^R}{\partial W_m^I} \cdot \Delta W_m^I + \frac{\partial S^R}{\partial R_m^R} \cdot \Delta R_m^R + \frac{\partial S^R}{\partial R_m^I} \cdot \Delta R_m^I
\]

\[
\frac{\partial S^I}{\partial W_n^R} \cdot \Delta W_n^R + \frac{\partial S^I}{\partial W_m^I} \cdot \Delta W_m^I + \frac{\partial S^I}{\partial R_m^R} \cdot \Delta R_m^R + \frac{\partial S^I}{\partial R_m^I} \cdot \Delta R_m^I
\]

Note that the noise is now regarded as those part of the data \(Z_{nm}\) which cannot be fitted by the model \(S\). The partial derivatives in (3c,d) are given by

\[
\frac{\partial S^R}{\partial W_n^R} = 1 - R_m^R, \quad \frac{\partial S^R}{\partial W_m^I} = R_m^I,
\]

\[
\frac{\partial S^R}{\partial R_m^R} = -W_n^R, \quad \frac{\partial S^R}{\partial R_m^I} = W_m^I
\]

\[
\frac{\partial S^I}{\partial W_n^R} = -R_m^I, \quad \frac{\partial S^I}{\partial W_m^I} = 1 - R_m^R,
\]

\[
\frac{\partial S^I}{\partial R_m^R} = -W_n^I, \quad \frac{\partial S^I}{\partial R_m^I} = -W_m^R
\]

Adopting a conventional nomenclature, these linearized equations are formulated in matrix notation as:

\[
B \cdot \mathbf{x} = \mathbf{y},
\]

where \(\mathbf{x}\) is the vector of model parameter updates \((\Delta W_{nm}^R, \Delta W_{nm}^I, \Delta R_{nm}^R, \Delta R_{nm}^I)\), \(\mathbf{y}\) comprises the reduced observations \((\Delta S_{nm}^R, \Delta S_{nm}^I)\), and \(B\) contains the partial derivatives \((4a)\). Both \(\mathbf{y}\) and \(B\) are evaluated at the initial model \((W_{nm0}^R, W_{nm0}^I, R_{nm0}^R, R_{nm0}^I)\). Continuity constraints on the reflectivity series \((C, \Delta \phi, \Delta \tau; \text{see next page})\) are appended in form of additional rows to \(B\) and \(\mathbf{y}\), such that (5) becomes

\[
\begin{bmatrix} B \\ C \end{bmatrix} \cdot \mathbf{x} = \begin{bmatrix} \mathbf{y} \\ -\Delta \tau \\ -\Delta \phi \end{bmatrix} = D \cdot \mathbf{x} = \mathbf{y}'
\]

Equation (6) is solved for \(\mathbf{x}\) by the least-squares solution:

\[
\mathbf{x} = (D^T \cdot D)^{-1} \cdot D^T \cdot \mathbf{y}'
\]

Although \(D^T \cdot D\) is over-determined, the inverse in (7) may be ill posed and is calculated by spectral decomposition. By allowing only large singular values and corresponding singular vectors to contribute to the solution, spectral decomposition minimizes the projection
of noise on the model in case of poorly conditioned systems. The columns of B are weighted to account for a possible numerical discrepancy between the source amplitude spectra and the reflectivity series spectra. Additionally, manually chosen weighting factors are introduced to determine the relative importance of source wavelets and reflectivity series. Due to the nonlinearity of equation (2), the updates are damped by a factor $\beta$ (<1) prior to their addition to the initial model:

$$[W^R, W^I, R^R, R^I] = \left[ W^R_0, W^I_0, R^R_0, R^I_0 \right] + \beta \cdot x$$ \hspace{1cm} (8)

$(W^R, W^I, R^R$ and $R^I$) in equation (8) are column vectors comprising the Fourier coefficients of all source wavelets and reflectivity series for a single frequency component. The choice of $\beta$ depends on the data. Finally, the initial model is replaced by the updated model, and the entire procedure is iterated starting from equations (3) until convergence is achieved. Stabilization of inversion can be achieved by adding additional constraints on the model parameters. E.g., asking for a smooth model by setting the second spatial derivative of the velocity to zero is common practice. Additionally, manually chosen weighting factors are introduced to determine the relative importance of source wavelets and reflectivity series. Due to the nonlinearity of equation (2), the updates are damped by a factor $\beta$ (<1) prior to their addition to the initial model:

$$\text{(9a)} \quad \Delta \tau_{AB} = \tau_A - \tau_B = \sqrt{(R^R_A + R^I_A)^2 - (R^R_B + R^I_B)^2} = 0$$

$$\text{(9b)} \quad \Delta \phi_{AB} = \phi_A - \phi_B = \tan^{-1} \left( \frac{R^I_A}{R^R_A} \right) - \tan^{-1} \left( \frac{R^I_B}{R^R_B} \right)$$}

The indices (A, B) are replaced by the correspond-

$C \cdot x = \begin{bmatrix} -\Delta \tau \\ -\Delta \phi \end{bmatrix}$ \hspace{1cm} (10)

Note that $x$ in equation (10) contains the real and imaginary parts of the reflectivity series ($\Delta R^R_m, \Delta R^I_m$) only, but not the source wavelets as in equation (5). The partial derivatives in $C$ calculate as:

$$\frac{\partial \Delta \tau_{AB}}{\partial R^R_A} = \frac{R^R_A}{\tau_A}, \quad \frac{\partial \Delta \tau_{AB}}{\partial R^I_A} = \frac{R^I_A}{\tau_A},$$

$$\frac{\partial \Delta \tau_{AB}}{\partial R^R_B} = -\frac{R^R_B}{\tau_B}, \quad \frac{\partial \Delta \tau_{AB}}{\partial R^I_B} = -\frac{R^I_B}{\tau_B},$$

$$\frac{\partial \Delta \phi_{AB}}{\partial R^R_A} = -\frac{R^I_A}{\tau_A}, \quad \frac{\partial \Delta \phi_{AB}}{\partial R^I_A} = \frac{R^R_A}{\tau_A},$$

$$\frac{\partial \Delta \phi_{AB}}{\partial R^R_B} = -\frac{R^I_B}{\tau_B}, \quad \frac{\partial \Delta \phi_{AB}}{\partial R^I_B} = -\frac{R^R_B}{\tau_B}$$ \hspace{1cm} (11a)

$$\frac{\partial \Delta \phi_{AB}}{\partial R^R_A} = -\frac{R^I_A}{\tau_A}, \quad \frac{\partial \Delta \phi_{AB}}{\partial R^I_A} = \frac{R^R_A}{\tau_A},$$

$$\frac{\partial \Delta \phi_{AB}}{\partial R^R_B} = -\frac{R^I_B}{\tau_B}, \quad \frac{\partial \Delta \phi_{AB}}{\partial R^I_B} = -\frac{R^R_B}{\tau_B}$$ \hspace{1cm} (11b)

The coefficients of $C$ and the misfits ($\Delta \tau, \Delta \phi$) are evaluated at each iteration step. The rows of $C$ are individually weighted by their interstation distance, and manually chosen weights control the importance of the amplitude continuity versus the phase continuity. After padding the source wavelets columns in $C$ and the source wavelet rows of $x$ with zeros, equations (10) are appended to $B$ and $y$ prior to the calculation of the inverse (equations 5 - 6).
Blind deconvolution of multichannel recordings

<table>
<thead>
<tr>
<th>Layer</th>
<th>Description</th>
<th>Upper - lower interface</th>
<th>Vp [km/s]</th>
<th>Vs [km/s]</th>
<th>Density [g/ccm]</th>
<th>Vertical extent (top / bottom)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>High-Velocity Layer (Carbonates)</td>
<td>Surface - low-angle fault</td>
<td>5.0</td>
<td>2.5</td>
<td>2.5</td>
<td>0 / 1-0</td>
</tr>
<tr>
<td>2</td>
<td>Sediment (Shale)</td>
<td>Surface/lower-angle fault - basement</td>
<td>4.0</td>
<td>1.96</td>
<td>2.3</td>
<td>1-0 / 4</td>
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<tr>
<td>3</td>
<td>Upper crust (Granite)</td>
<td>Basement - crustal interface</td>
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<td>3.7</td>
<td>2.8</td>
<td>4 / 20</td>
</tr>
<tr>
<td>4</td>
<td>Lower crust</td>
<td>Crustal interface - Moho</td>
<td>6.5</td>
<td>3.71</td>
<td>2.85</td>
<td>20 / 37</td>
</tr>
<tr>
<td>5</td>
<td>Mantle</td>
<td>Moho</td>
<td>8.0</td>
<td>4.45</td>
<td>3.3</td>
<td>37</td>
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</table>

Table 1. Parameters of the synthetic model (Figure 2).

<table>
<thead>
<tr>
<th>Reflection / Transmission</th>
<th>From layer 1</th>
<th>From layer 2</th>
<th>From layer 3</th>
<th>From layer 4</th>
<th>From layer 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>To layer 1</td>
<td>–</td>
<td>0.15/0.85</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>To layer 2</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.31/1.31</td>
<td>0.15/1.15</td>
</tr>
<tr>
<td>To layer 3</td>
<td>–</td>
<td>0.31/0.69</td>
<td>–</td>
<td>–</td>
<td>0.02/1.02</td>
</tr>
<tr>
<td>To layer 4</td>
<td>–</td>
<td>–</td>
<td>0.02/0.98</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>To layer 4</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.18/1.18</td>
<td>0.18/1.18</td>
</tr>
</tbody>
</table>

Table 2. Vertical incidence reflection/transmission coefficients for the synthetic model (Figure 2).

3 SYNTHETIC TESTS

The applicability of the outlined methodology is tested on a synthetic data set. A 2D reflectivity model is based on the expected subsurface structure of the region where the field data (cf. section 4) were acquired (Leahy et al., 2012; Gans, 2011). It comprises a 9 km long section with 5 distinct layers (Figure 2, Table 1). With the exception of the low-angle fault, which separates layer 1 from layer 2, all interfaces are horizontal. Transmission and reflection coefficients between the layers are calculated for vertical incidence P-waves (Aki and Richards, 2009) and are summarized in Table 2. Fifty-five receivers are distributed along the 9 km long section. For each receiver, a reflectivity model r(t) is obtained by multiplication of the according reflection and transmission coefficients for the down- and upgoing waves and conversion to vertical two-way travel times. These total reflectivity coefficients calculate as -0.15, 0.30, 0.02 and 0.16 for the low-angle fault, the basement, the upper/lower crust boundary and the Moho at the left side of the section where the low-angle fault is present. The corresponding numbers at the right side, where the low-angle fault does not exist, are almost identical (0.31, 0.02 and 0.16 for the basement, the upper/lower crust boundary, and the Moho). Five source wavelets w(t) are calculated by applying different Butterworth filters to a spike (Ryan, 1994), and synthetic observation data z(t) for each wavelet are derived by evaluating equation (1). The Butterworth filter is chosen for its flexibility in creating minimum- and mixed-phase spectra which also characterize real earthquake wavelets. The used frequency content ranges from 0.3 to 1.8 Hz. Those values where selected to resemble the real data set (cf. section 4). White noise is added to z(t) in the frequency range 0.2 - 2 Hz, where the signal-to-noise (S/N) ratio is specified as the ratio of the maximum absolute amplitude of w(t) to the maximum range of the uniform noise distribution. The used wavelets and examples of synthetic data are shown in Figure 3. The synthetic data illustrate the two major challenges that are (i) the burial of shallow reflections beneath the source wavelet and (ii) the weak reflection signatures in presence of noise. If the source wavelet is of more complex shape (e.g., wavelet no. 2), these difficulties aggravate. A multitude of inver-
Figure 3. Five different source wavelets and resulting synthetic data for source wavelet no. 5. Dashed black lines indicate the vertical two-way travel times associated with the reflectors (fault, basement, upper/lower crust boundary, Moho) shown in Figure 2.

Additional information about data processing and inversion parameters. Inversions of this synthetic data set are performed in order to gain a better understanding of the inversion parameters. Five source wavelets and 55 stations result in 275 observed traces versus 60 unknown source wavelets and reflectivity series. The exact onset of a wavelet is usually hard to define on real data, but it is not needed to be known since the phase spectra is part of the solution. The initial reflectivity is zero at all stations, and the initial source wavelets are calculated as the average wavelet of the first 6 seconds of each event. Apart from noise-free data, two additional data sets with S/N ratios of 2.0 and 1.0, respectively, are also inverted. For comparison, conventional single-trace deconvolution including stacking over all events is also applied to the data. The estimated source wavelet for deconvolution is identical to the initial source wavelet for blind deconvolution. In the following, I summarize the most important findings of the synthetic tests.

The number of used singular values and singular vectors depend on the signal-to-noise ratio, but also on the weight of the continuity constraints. In more or less all inversions, the damping factor $\beta$ (equation 8) must be kept small (0.1 - 0.2) to guarantee stability, which indicates a strong degree of nonlinearity. Accordingly, a large number of iterations (30 - 50) are necessary to achieve convergence. A crucial parameter is the relative weight of the source wavelet and the reflectivity series (Figure 4). Although the effect of strong vs. weak weighting has not much effect on the estimation of the source wavelet itself (Figures 4d, e), there is a pronounced difference in the obtained reflectivity series (Figures 4a, b). The overall ringy character of the reflectivity solution shown in Figure 4b possibly expresses a compensation for spurious small amplitudes in the corresponding source wavelet solution (Figure 4e). On the other hand, only the result from strong source wavelet weighting (Figure 4b) indicates the reflection from the low-angle fault. Thus, there is no obvious best choice for this parameter, and I take this as another expression of strong nonlinearity. The true, initial and obtained source wavelets are very similar in general, which possibly results from the strong amplitude of the incoming wave in comparison to the reflected waves. Conventional deconvolution applied to the noise-free data set (Figure 4c) provides a result similar to blind deconvolution with weak source wavelet weighting (Figure 4a). However, more artifacts appear in the uppermost section (0 - 1 s) since the deconvolution does not take the superposition of the incoming and reflected waves into account. The weak reflectivity contrast at the upper/lower crust boundary is not resolved adequately with any of the methods. It is also noted that the polarity of the reflectors is correctly reconstructed. Fig. 5 illustrates results from the inversion of the data set with a signal-to-noise ratio of 1. There is a clear improvement when the continuity constraints are enforced (Figure 5b vs. Figure 5a). In opposite, conventional deconvolution (Figure 5c) performs poorly in reconstructing the reflectivity series. The low-angle fault is not recovered reliably in any of the inversions nor deconvolutions. A second suite of tests was run on synthetic data sets based on more complicated source wavelets (e.g., longer coda). As expected, the recovery of the reflectivity structure degrades with the complexity of the source wavelets, in
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Figure 4. Estimated reflectivity time series and source wavelets from noise-free synthetic data: Reflectivity series (a) and source wavelets (d) for weak (0.1) relative source wavelet weighting. Reflectivity series (b) and source wavelets (e) for strong (1.0) relative source wavelet weighting. Note the spurious amplitudes at later times ($T > 12$ s) in (e). (c) Reflectivity series obtained by deconvolution (reversed polarity). (f) Initial source wavelets for inversion and deconvolution. (g) True source wavelet used for generation of synthetic data. All traces are bandpass filtered in the range of 0.5 - 2 Hz. Dashed grey lines indicate vertical two-way travel times associated with the reflectors in the synthetic model.

particular in combination with a low signal-to-noise ratio. In this case, the continuity constraints become more important, and blind deconvolution performs far superior to standard deconvolution.

4 APPLICATION TO FIELD DATA

The La Barge seismic experiment is an industry-academia cooperation and aims at evaluating the use of low-frequency passive seismic data for local sub-surface characterization (Saltzer et al., 2011). From November 2008 to June 2009, fifty-five 3-component broadband instruments (Guralp CMG 3T, natural period 120 s) were continuously recording over an active oil/gas production site in Wyoming (Figure 6). The site is located in the La Barge region in the Green River Basin. On a local scale, the dominant structural feature is the low-angle Hogback thrust which separates claystones and siltstones in the east from an approximately 1 km thick carbonate sequence in the west. The instruments were deployed at the surface with a very narrow station spacing of 250 m, and the sample interval was 10 milliseconds. Data from the La Barge seismic experiment were
subject to several other studies. Gans (2011) calculated P-to-S receiver functions for illumination of the entire crust and uppermost mantle and found remarkable variations on a very small spatial scale. Leahy et al. (2012) were able to derive comparably high-resolution receiver function images of the shallow crust by exploiting the upper frequency range of teleseismic events. Biryol et al. (2013) retrieved velocity models of the shallow crust by finite-frequency travel time inversion of regional seismic events. Behm et al. (2013) used traffic noise from a nearby road to obtain local surface wave velocities. While all the applied methods have their origin in earthquake seismology, the novelty of most of these studies is their focus on the uppermost part of the crust (<5 km depth).

4.1 Teleseismic events

Due to the long recording interval, a large number of teleseismic events were collected. The assumption of vertical incidence implies the use of events with an epicentral distance greater than 25 degrees, which in turn requires magnitudes greater than 5.5 for a clear observation. The strength of a particular teleseismic phase depends on the source mechanism and on the geologic structure at the source location. A well defined wavelet may only be obtained if different phases of a single event
Figure 6. Location of the 55 broadband stations of the La Barge Passive Seismic Experiment. The Hogsback thrust separates carbonates in the west from claystones and siltstones in the east.

<table>
<thead>
<tr>
<th>No.</th>
<th>Usage</th>
<th>Date/Time</th>
<th>Mag.</th>
<th>Distance [deg.]</th>
<th>BAZ [deg.]</th>
<th>Long. [deg.]</th>
<th>Lat. [deg.]</th>
<th>Depth [km]</th>
</tr>
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<td>95.0</td>
<td>251.7</td>
<td>168.47</td>
<td>-17.14</td>
<td>205</td>
</tr>
<tr>
<td>2</td>
<td>P,pP</td>
<td>2008-11-21 / 07:05:35</td>
<td>6.1</td>
<td>96.0</td>
<td>263.6</td>
<td>159.55</td>
<td>-8.95</td>
<td>263</td>
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<tr>
<td>3</td>
<td>pP</td>
<td>2009-03-15 / 08:19:05</td>
<td>5.7</td>
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<td>-70.36</td>
<td>-14.45</td>
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<td>-21.48</td>
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<td>5</td>
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<td>2009-04-18 / 02:03:53</td>
<td>5.8</td>
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<td>-28.92</td>
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<td>6</td>
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<td>2009-04-18 / 19:17:59</td>
<td>6.6</td>
<td>65.8</td>
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<td>151.43</td>
<td>46.01</td>
<td>35</td>
</tr>
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<td>7</td>
<td>P</td>
<td>2009-04-21 / 05:26:12</td>
<td>6.2</td>
<td>61.1</td>
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<td>155.01</td>
<td>50.83</td>
<td>152</td>
</tr>
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<td>8</td>
<td>P</td>
<td>2009-05-22 / 00:24:21</td>
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<td>33.1</td>
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<td>9</td>
<td>P</td>
<td>2009-05-22 / 19:24:19</td>
<td>5.6</td>
<td>26.2</td>
<td>153.8</td>
<td>-98.46</td>
<td>18.11</td>
<td>62</td>
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(e.g. direct arrival (P), reflection from the outer core (PcP), source-side surface reflection (pP), midpoint surface reflection (PP)) are well separated in time. The degree of separation depends on the focal mechanism, on the distance and on the depth of the event. As some of these factors may be poorly known and/or weakly constrained, a visual inspection of a range of pre-selected event gathers is still necessary. The final selection criteria are the requirements of a short duration pulse of the source wavelet and the absence of later phases within a time range according to the investigation depth. I use both direct arrivals (P) and source-side surface reflections (pP) of 9 different events as source wavelets (Table 3). These wavelets have main frequencies between 0.4 and 1.2 Hz.

4.2 Data and pre-processing
As this study focuses on P-waves, only vertical component data are used subsequently. The data are band-
Figure 7. Examples of vertical component data used for the inversion (events 2 and 8, Table 3). P and pP refer to the teleseismic phases (P: direct arrival, pP: source-side surface reflection). Upper row: Band-pass filtered (0.6 - 4 Hz) event gathers. Time zero is approximately 2 seconds prior to the onset of the phase at the westernmost station. The time delay of the easternmost stations is due to their larger distances to the array. Note the converted surface waves ("cR") arriving at times greater than 8 seconds. Lower row: Corresponding time-aligned and singular-value decomposed event gathers which are the input data for inversion and deconvolution.

pass filtered between 0.6 - 4 Hz, and they are cut to a length of 30 seconds, starting one second prior to the earliest onset of the used phase (Figure 7, upper row). The traces are then aligned by their difference in arrival time to the receiver with the earliest onset. Arrival time picking is done manually, since the estimation of the reflectivity series is not significantly flawed by small inaccuracies with respect to the used frequency range. The same accounts for the calculation of the initial source wavelet from the aligned traces. To avoid numerical issues in the inversion due to different magnitudes, all event gathers are normalized to their maximum amplitude. An analysis of the entire data set suggests the presence of strong converted surface waves associated with the majority of the events (phase "cR" in Figure 7). Conversions from incident teleseismic body waves to Rayleigh waves are attributed to lateral Moho variations or rugged topography (e.g. Neele and Snieder (1991)). Their directionality indicates an origin in the west to northwest of the deployment, and their arrival times suggest conversion distances greater than 30 kilometers. The southern part of the Wyoming Mountain Range is located in this area and it is interpreted to account for the conversions, which occur as early as 6 seconds after the onset of the body waves. As reflections from the lower crust and the Moho are expected to arrive between 8 and 14 seconds, the superposition of the converted surface waves poses an additional challenge. Singular value decomposition (SVD) of a wave field is a practical way to attenuate or enforce laterally coherent features of event gathers (Freire and Ulrych, 1988). By choosing a limited range of the largest singular values...
and corresponding singular vectors, laterally coherent signals are amplified at the expense of dipping phases and high-frequency noise. The choice of the range of singular values is somewhat subjective, and selecting too few singular values might result in suppressing laterally varying subsurface structures. Tests showed that in case of the P-phase the inclusion of the upper 15% of the singular values and singular vectors provides a significant improvement of the inversion result without degrading the main structural features of the inverted reflectivity series. The corresponding number for the pP-phase is 40%.

### 4.3 Inversion and results

Inversions are carried out for P and pP data (Table 3), and the results are shown in Figure 8. Inversion parameters (Table 4) are chosen based on the insights from the synthetic tests. I further use conventional deconvolution as well as results from teleseismic travel time inversion (Biryol et al., 2013) and receiver function analysis (Gans, 2011; Leahy et al., 2012) to validate my findings.

All inversions derive a continuous reflector with positive polarity at a time of approximately 2 seconds. The arrival times are slightly less in the western part. The results from the pP data (Figure 8c) are shifted by about 0.3 seconds compared to P data. This shift might be related to the lower frequency of the pP wavelets, and should be subject to further analysis. Enforcement of the continuity constraints (Figure 8b) facilitates the recovery of a reflector at about 14 s in the P dataset. The result obtained from conventional deconvolution (Figure 8d) appears noisier overall, and coherent later arrivals are more difficult to interpret. The inversion from P data with enforced continuity constraints (Figure 8b) is converted to depth with a 1D velocity-depth-model based on the aforementioned studies (Figure 9). The shallow reflector at approximately 4 km depth correlates well with the basement transition from low-velocity sandstones to high-velocity carbonates and granites. The small but significant apparent rise of the reflector in its western part is attributed to the uppermost thin sheet of carbonates prevailing only west of and above the low-angle Hogsback thrust. The higher velocities of the carbonates are not included in the 1D velocity model and result in an apparent shallower reflector. The Hogsback thrust itself cannot be deduced reliably from the inversion, but a pronounced lateral change of the reflectivity is observed in the according depth range. Gans (2011) interpreted a seismic discontinuity at ca. 15 km depth but this is not indicated by the blind deconvolution inversion. The reason for this disagreement is not entirely clear, but it might be due to a combination of weak P-wave impedance contrast and superposition of converted surface waves in the blind deconvolution data. The reflections from ca. 40-42 km depth correlate well with the Moho depth range given by Gans (2011). The strong apparent dip (8 degrees) towards the west is surprising, but Gans (2011) also found the Moho depths to be about 2 km deeper west of the Hogback thrust than in the eastern part of the array. Considering the small array aperture, the enforced continuity constraints might smear an abrupt lateral change into a dipping structure. Lateral velocity changes above the Moho may also partly explain the apparent dip.

### 5 DISCUSSION

I start with comparing the presented inversion scheme to conventional deconvolution. The main difference is that the reflectivity series now is described by a model with which all observations must comply, while conventional (single-trace) deconvolution derives a new model for each seismic event. This drawback might be overcome by stacking deconvolved traces for several events, but there still is a strong sensitivity on noise. Modeling by inversion provides a more natural way to minimize the influence of white noise, although coherent noise over several traces (e.g. surface waves arriving at oblique angles, or later arriving vertical-incidence waves) can be projected into the reflectivity model. The simultaneous estimation of the wavelet and the reflectivity series is considered as an advantage over methods which calculate the wavelet only prior to the reflectivity series. As with any inversion, constraints on the model and the determination (weights) of the unknowns can be incorporated in a flexible and natural way. The superposition with the source wavelet is an important modification to the conventional deconvolution model in case of shallow structures, in particular when the S/N-ratio is low (e.g. Figures 4, 5). Application to both synthetic and real data shows that the inversion scheme is more stable than conventional deconvolution. White noise is handled well by the inversion, while coherent noise over several traces presents a challenge, and such signals must be eliminated by pre-processing. If the cause of the coherent noise is well known, it may also be modeled and estimated in the inversion. The assumption of vertical incidence presents a severe constraint to the method. Teleseismic events with epicentral distances greater than 25 degrees and thus large magnitudes are required, and the need for clear and well-defined source wavelets imposes a strong selection criterion. In practice, it implies a long observation period to collect a sufficient number of events. In this study, the frequency range of the teleseismic phases is confined to the range of 0.1 to 1.2 Hz which limits the resolution capabilities. However, no special efforts were made to exploit the higher frequency range (5 - 10 Hz) of earthquake body waves (e.g. Leahy et al. (2012). Advanced pre-processing in future studies should also focus on this aspect. The assumption of a stationary source wavelet restricts the lateral extent of the deployment in general, but the actual aperture depends on the specific subsurface characteristics of a
Figure 8. Reflectivity series obtained from the La Barge data. (a) Reflectivity series from P phases without applying continuity constraints. (b) Reflectivity series from P phases with enforced continuity constraints. (c) Reflectivity series from pP phases without applying continuity constraints. (d) Conventional deconvolution of P wavelets. The dashed grey line depicts the maximum amplitude of the shallow reflector in (b), and it is superimposed on the other panels for comparison. See text for details.
Table 4. Parameter used for the inversion results shown in Fig. 8. The SVD ratio $\alpha$ describes the threshold for singular values and vectors. Only those singular values, whose ratio to the largest singular value are higher than $\alpha$, are used in the inversion.

<table>
<thead>
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<th>Fig.</th>
<th>Used phase / no. of events</th>
<th>SVD ratio $\alpha$</th>
<th>Damping factor $\beta$ / no. of iterations</th>
<th>Source weight</th>
<th>Initial wavelet length [s]</th>
<th>Continuity constraints</th>
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<td>8a</td>
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<td>0.1 / 50</td>
<td>0.15</td>
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<td>0.1 / 50</td>
<td>0.10</td>
<td>5.3</td>
<td>No</td>
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Figure 9. Depth converted seismic image from Figure 8b. The 1D velocity-depth model is based on previous studies (Gans, 2011; Leahy et al., 2012; Biryol et al., 2013). The cartoon including the geological section and the velocity model is taken from Biryol et al. (2013). The dashed black line in the seismic images indicates the Hogsback thrust as known from surface geology and active source seismic investigations. See text for details.

region in question. A laterally homogenous medium is more advantageous, and applications to other data sets will help to obtain a better understanding of the scale range of the method. The multichannel approach is not limited to specific deployment geometry and thus well suited for common passive seismic experiments on a local scale. Larger recording arrays may be targeted by allowing the source wavelet to vary spatially through interpolation, but this remains a subject to further studies. The presented scheme can be used to supplement conventional receiver function methods due to the similar type of input data, but it does not require three-component observations. However, structural information on the shear-wave reflectivity might be additionally obtained by evaluating shear body waves on the horizontal components. With respect to the targeted depth range and resolution capabilities, low-frequency geophones might be used in future deployments to sub-
stitute for costly broadband stations. The focus of the presented study is the inversion for local and shallow reflectivity series, and the inversion parameters are chosen in such a way to derive a realistic subsurface model. This comes at the expense of projecting noise into the source wavelets. Future research should be dedicated to the applicability of the method for exact source wavelet reconstruction (e.g., by assuming realistic initial reflectivity models or advanced pre-processing).

6 CONCLUSION
With respect to the initial question on the applicability for exploration seismology, the outlined method presents a potentially useful technique for local large-scale subsurface characterization from passive seismic data. Shallow to intermediate and deep crustal structures can be imaged from a comparably low number of (well-suited) teleseismic earthquakes. As with any inversion technique, the results are sensitive to inversion parameters and the input data. Nonetheless, synthetic and real data examples indicate improved performance in comparison to conventional deconvolution which is attributed to a common model for all observations. Further studies, and in particular applications to other datasets will help to develop a better understanding of the merits and practical limitations of the method.

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