Multiparameter TTI tomography of P-wave reflection and VSP data

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ABSTRACT
Transversely isotropic models with a tilted symmetry axis (TTI media) are widely used in depth imaging for complex geologic structures. Here, we present a modification of a previously developed 2D P-wave tomographic algorithm for building heterogeneous TTI models and apply it to synthetic data. The symmetry-direction velocity $V_{P0}$, anisotropy parameters $\epsilon$ and $\delta$, and the symmetry-axis tilt $\nu$ are defined on a rectangular grid. To ensure stable reconstruction of the TTI parameter fields, reflection data are combined with walkaway VSP (vertical seismic profiling) traveltimes in joint tomographic inversion. To improve the convergence of the inversion algorithm, we propose a three-stage model-updating procedure that gradually relaxes the constraints on the spatial variations of the anisotropy parameters, while the symmetry axis is kept orthogonal to the reflectors. Only at the final stage of the inversion the parameters $V_{P0}$, $\epsilon$, and $\delta$ are updated on the same grid. We also incorporate geologic constraints into tomography by designing regularization terms that penalize parameter variations in the direction parallel to the interfaces. First, the reflection tomography without borehole constraints is tested on a model that includes a bending TTI layer with a wide range of dips. Then we examine the performance of the regularized joint tomography of reflection and VSP data for two sections of the BP TTI model that contain an anticline and a salt dome. The TTI parameters in the shallow part of both sections (down to 5 km) are well-resolved by the three-stage model-updating process. Due to the limited constraints from reflection events and sparse coverage of VSP rays at depth, the velocity field in the deeper part of the section is estimated with larger errors. These results provide useful guidance for building accurate TTI models for prestack depth imaging.

1 INTRODUCTION

 Prestack depth imaging for complex geologic environments (including fold-and-thrust belts and subsalt plays) requires anisotropic velocity models such as transverse isotropy with a vertical (VTI) or tilted (TTI) axis of symmetry. Therefore, conventional isotropic reflection tomography for isotropic media (Stork, 1992) need to be extended to heterogeneous anisotropic media (Campbell et al., 2006; Woodward et al., 2008). Because P-wave reflection traveltimes do not provide sufficient constraints for resolving all relevant TTI parameters, it is necessary to use additional information (e.g., borehole data) to reduce the nonuniqueness of the inverse problem (Morice et al., 2004; Tsvankin, 2005; Bakulin et al., 2010a).

P-wave velocity in TTI media is controlled by the velocity $V_{P0}$ in the symmetry-axis direction, anisotropy parameters $\epsilon$ and $\delta$, and the orientation of the symmetry axis (in 2D defined by the tilt $\nu$ from the vertical).

Zhou et al. (2011) develop multiparameter reflection tomography for TTI media and apply it to synthetic and field data. They find that simultaneous estimation of all three relevant parameters ($V_{P0}$, $\epsilon$, and $\delta$) helps flatten the common-image gathers (CIGs) better than single-parameter (only $V_{P0}$) inversion. Zhou et al. (2011) also confirm that trade-offs between the TTI parameters cannot be eliminated using only P-wave reflections, and point out the importance of additional constraints from well data. However, they do not carry out joint inversion of reflection and borehole data by including, for example, VSP (vertical seismic profiling) traveltimes.

Nonuniqueness of the inversion of reflection data can be mitigated by regularization which imposes a priori constraints on the estimated model (Engl et al., 1996). Wang and Tsvankin (2011; hereafter, referred to as Paper I) employ Tikhonov (1963) regularization to smooth the velocity field both horizontally and vertically. Fomel (2007) develops so-called “shaping regular-
ization" designed to steer the velocity variations along geologic structures (e.g., layers). Its mapping (shaping) operator is integrated into the conjugate-gradient iterative solver. Using the steering-filter preconditioner (Clapp et al., 2004) similar to shaping regularization, Bakulin et al. (2010b) perform joint tomographic inversion of P-wave reflection data (from horizontal and dipping reflectors) and check-shot traveltimes for VTI media. They conclude that in the vicinity of the well it is possible to resolve the vertical variation of all three relevant parameters \( (V_{\text{foc}}, \epsilon, \text{and} \delta) \).

In Paper I, we develop a 2D ray-based tomographic algorithm for iteratively updating the parameters \( V_{\text{foc}}, \epsilon, \text{and} \delta \) of TTI media defined on rectangular grids (the symmetry axis is set orthogonal to the imaged reflectors). Synthetic tests for a simple model with a "quasi-factorized" TTI syncline (i.e., \( \epsilon \) and \( \delta \) are constant inside the TTI layer, while the tilt \( \nu \) may vary spatially) demonstrate that stable parameter estimation requires strong smoothness constraints or additional information from walkaway VSP traveltimes.

Here, we first introduce the objective function that contains the residual moveout in CIGs and the VSP traveltimes misfit supplemented by regularization terms. The regularization is designed to smooth the parameters in the direction parallel to the interfaces, while allowing for more pronounced variations in the orthogonal direction. Next, we present a three-stage inversion methodology in which gridded tomography is preceded by two partial parameter-updating steps designed to stabilize the inversion. Then the tomographic algorithm is tested on a model that contains a TTI thrust sheet, and on two sections of the 2D TTI model devised by BP.

## 2 METHODOLOGY

The tomographic algorithm employed here is described in detail in Paper I. The residual moveout in common-image gathers produced by Prestack Kirchhoff depth migration is minimized by iterative parameter updates. If walkaway VSP surveys (check shots represent zero-offset VSP data) are available, VSP traveltimes are computed for each trial model and included in the objective function:

\[
F(\Delta \lambda) = \| A \Delta \lambda - b \|^2 + \epsilon_{VSP}^2 \| E \Delta \lambda - d \|^2 + R(\Delta \lambda),
\]

where the vector \( \Delta \lambda \) represents the parameter updates, the elements of the matrix \( A \) are the traveltime derivatives with respect to the medium parameters at each grid point (\( A \) is computed analytically along the raypaths), \( b \) is a vector containing the residual moveout in CIGs, the matrix \( E \) is composed of VSP traveltime derivatives, and the vector \( d \) is the difference between the observed and calculated VSP traveltimes for each source-receiver pair. The regularization term \( R \) in equation 1 has the form:

\[
R(\Delta \lambda) = \zeta^2 \| \Delta \lambda \|^2 + \zeta_1^2 \| L_1(\Delta \lambda + \lambda_0) \|^2 + \zeta_2^2 \| L_2(\Delta \lambda + \lambda_0) \|^2,
\]

where \( \zeta^2 \| \Delta \lambda \|^2 \) restricts the magnitudes of parameter updates that have small derivatives in the matrices \( A \) and \( E \), the operators \( L_1 \) and \( L_2 \) are designed to make parameter variations more pronounced in the direction normal to the interfaces, and \( \zeta, \zeta_1 \), and \( \zeta_2 \) are the regularization coefficients/weights. In 2D, the normal direction of a reflector is defined by the dip angle with the vertical, which is computed from the depth image using Madagascar program "sfdip." Then the symmetry-axis tilt \( \nu \) at each grid point is set to be equal to the corresponding dip.

To construct the matrix \( L_1 \), we first compute two components of the gradient vector \( \nabla \lambda \) from the following finite-difference approximation:

\[
\lambda'(x) = \left[ \frac{\lambda(x + dx) - \lambda(x - dx)}{2dx} + O((dx)^2) \right],
\]

\[
\lambda'(z) = \left[ \frac{\lambda(z + dz) - \lambda(z - dz)}{2dz} + O((dz)^2) \right],
\]

where \( \lambda \) is the parameter \( (V_{\text{foc}}, \epsilon, \text{or} \delta) \) at the grid point with the coordinates \( x \) and \( z \), and \( dx \) and \( dz \) are the cell dimensions. Since the dip field yields the vector \( n \) orthogonal to reflectors, we minimize the norm of the cross-product \( \| n \times \nabla \lambda \| \) at all grid points, which is equivalent to aligning the direction of the largest parameter variation with \( n \) and restricting the variations along interfaces. To include more cells, we can use a higher-order finite-difference approximation:

\[
\lambda'(x) = \left[ \frac{-\lambda(x + 2dx) + 8\lambda(x + dx) - 8\lambda(x - dx) + \lambda(x - 2dx)}{12(dx)^2} + O((dx)^4) \right],
\]

\[
\lambda'(z) = \left[ \frac{-\lambda(z + 2dz) + 8\lambda(z + dz) - 8\lambda(z - dz) + \lambda(z - 2dz)}{12(dz)^2} + O((dz)^4) \right].
\]

Similarly, the finite-difference approximation of the Laplacian operator \( \nabla^2 \lambda \) has two components:

\[
\lambda''(x) = \left[ \frac{\lambda(x + dx) - 2\lambda(x) + \lambda(x - dx)}{(dx)^2} \right],
\]

\[
\lambda''(z) = \left[ \frac{\lambda(z + dz) - 2\lambda(z) + \lambda(z - dz)}{(dz)^2} \right].
\]

Then we compute the cross-product \( n \times \nabla^2 \lambda \) and minimize its norm to mitigate the parameter variations in the direction parallel to the interfaces.

Defining the TTI parameters on a relatively small grid results in a large number of unknowns, and the Fréchet matrices \( A \) and \( E \) in equation 1 are sparse (i.e., only nonzero or large elements are stored due to the limited computer memory). To solve the large sparse linear system of equations in an efficient way, we employ the parallel direct sparse solver (PARDISO) from Intel Math Kernel Library (MKL, http://software.intel.com/en-us/articles/intel-mkl/).
As described in Paper I, the anisotropic velocity field is iteratively updated starting from an initial model that may be obtained from stacking-velocity tomography at borehole locations (Wang and Tsvankin, 2010). In the first few iterations, the symmetry-direction velocity \( \nu_0 \) is typically inaccurate and simultaneous inversion for all TTI parameters may result in unacceptably large updates for the anisotropy parameters \( \epsilon \) and \( \delta \). If the anisotropy parameters are moderate (\( \epsilon < 0.25 \) and \( \delta < 0.15 \) in our tests), it is convenient to fix them temporarily at the initial values (typically small) and limit the updates to the velocity \( \nu_0 \). At the second stage of parameter updating, the model is divided into several layers based on the picked reflectors, and the anisotropy parameters are assumed to be constant within each layer. The velocity \( \nu_0 \) is then updated on a grid, while \( \epsilon \) and \( \delta \) change in each layer (i.e., the inversion for \( \epsilon \) and \( \delta \) is layer-based). Such a “quasi-factorized” assumption is equivalent to strong smoothing of the anisotropy parameters and may help resolve all TTI parameters if \( \nu_0 \) is a linear function of the spatial coordinates and at least two distinct dips are available (Behera and Tsvankin, 2009). At the third and last stage of velocity analysis, the parameters \( \epsilon \) and \( \delta \) are updated on the same grid as that for \( \nu_0 \) to allow for more realistic treatment of heterogeneity. Still, because P-wave kinematics are less sensitive to the anisotropy parameters than to the symmetry-direction velocity, the \( \epsilon \)– and \( \delta \)-fields in function 2 should be regularized with larger weights.

3 SYNTHETIC EXAMPLES

3.1 TTI thrust sheet

First, the tomographic algorithm is tested on the synthetic data of Zhu et al. (2007), whose model (simulating typical structures in the Canadian Foothills) includes a TTI thrust sheet embedded in an otherwise isotropic, homogeneous medium (Figure 1). P-wave reflection data were generated by an anisotropic finite-difference code. We used sources and receivers placed every 60 m with the maximum offset reaching 1980 m. Because the exact model geometry (i.e., the interface positions) is unavailable, we cannot provide comparisons of our migration results with the correct model.

The initial model for reflection tomography includes two horizontal isotropic layers (Figure 2a). Although the P-wave velocity in the isotropic background is set to the correct value, ignoring transverse isotropy in the bending layer causes noticeable residual moveout in common-image gathers (Figure 2b) and a strong distortion of the imaged reflector beneath the thrust sheet (Figure 2c). In the velocity-updating process, the velocity \( \nu_0 \) is defined on a square (100 m \( \times \) 100 m) grid.

Because of the relatively simple model geometry, it is not necessary to follow the three-stage inversion procedure introduced above. Based on the picked reflectors, the section is divided into two isotropic blocks and a TTI layer sandwiched between them. Each layer/block is assumed to be “quasi-factorized” TTI with constant anisotropy parameters \( \epsilon \) and \( \delta \) (i.e., we only use the second step of the updating procedure). The symmetry axis is taken perpendicular to the reflectors, with the tilt changing during the updates. Because there is only a single horizontal reflector on the right side of the model (Figure 1), the parameters \( \epsilon \) and \( \delta \) above it cannot be resolved solely from P-wave reflection data. Therefore, both \( \epsilon \) and \( \delta \) in the block to the right of the TTI layer are set to zero.

With a general smoothing operator (Wang and Tsvankin, 2011) instead of \( L_1 \) and \( L_2 \) in function 2, the velocity in the TTI layer is partially recovered (Figure 3a) after 12 iterations. Because \( \nu_0 \) is updated on a relatively fine grid, flattening the CIGs along three reflectors (and just one reflector for the block on the right side of the model) is insufficient for recovering the velocity field, even when regularization is applied. For example, there is noticeable heterogeneity in each block that does not exist in the correct model.

Nevertheless, the constraints provided by a wide range of dips in the TTI thrust sheet and the correct assumption about the spatial variation of \( \epsilon \) and \( \delta \) helps accurately resolve both anisotropy parameters (Figure 3b and 3c). The error in \( \epsilon \) in the TTI layer (0.03) is somewhat larger than that in \( \delta \) (-0.01) because of the small offset-to-depth ratio, which is close to unity for the bottom reflector. As shown by Behera and Tsvankin (2009), stable estimation of \( \epsilon \) in quasi-factorized TTI media requires long spreads reaching two reflector depths. Despite remaining distortions in \( \nu_0 \), the obtained anisotropic velocity field largely removes the residual moveout in the CIGs (Figure 4a) and improves the depth image, especially that of the bottom horizontal reflector (Figure 4b). Additional reflectors or
walkaway VSP data would help refine the velocity field and obtain a more accurate spatial distribution of $V_{P0}$. Another way to suppress spurious spatial variations of $V_{P0}$ is to apply stronger smoothness constraints, which could be based on a priori information about the model.

### 3.2 BP anticline model

Next, we test the joint tomography of P-wave long-spread reflection data and walkaway VSP traveltimes on a section of the BP TTI model that contains an anticline structure (http://www.freeusp.org/2007_BP_Ani_Vel_Benchmark/). The velocity $V_{P0}$ in the actual model is smoothly varying (Figure 7a), except for a small jump at the water bottom. The symmetry axis is set perpendicular to the interfaces (Figure 7b), and the anisotropy parameters $\epsilon$ and $\delta$ change from layer to layer with relatively weak lateral variations compared to those of $V_{P0}$ (Figure 7c and 7d). The depth image produced by Kirchhoff prestack depth migration with the correct velocity model is shown in Figure 8.

Migration velocity analysis (MVA) is applied to CIGs from $x = 40$ km to 61 km with an interval of 150 m (the maximum offset is 10 km). Synthetic VSP data were generated in a vertical “well” placed at location $x_{VSP} = 51.4$ km with 24 receivers spanning the interval from $z = 4756.25$ m to 5043.75 m every 12.5 m and 24 more receivers evenly placed between 7837.5 m and 8125 m. The VSP sources were located at the
Figure 4. (a) CIGs after 12 iterations and (b) the corresponding depth image obtained with the parameters from Figure 3. Note that the reflector beneath the thrust sheet has been mostly flattened.

To build an initial model, we compute a 1D profile of \( V_{P0} \) from the check-shot traveltimes and then obtain the 2D velocity field (Figure 5) by extrapolation that conforms to the picked interfaces. The exact position of the water bottom is assumed to be known, and the velocity of the water layer is fixed at the correct value.

The noticeable residual moveout in the CIGs (Figure 9a) and the distorted depth image (Figure 9b) indicate that the velocity field contains significant errors.

Following the three-stage parameter-estimation procedure described above, first we update only the velocity \( V_{P0} \) defined on a rectangular 200 m \( \times \) 100 m grid, while keeping the anisotropy parameters \( \epsilon \) and \( \delta \) set to zero. Then \( \epsilon \) and \( \delta \) are taken constant in each layer (delineated by the interfaces picked on the image), but updated simultaneously with the velocity. With this “quasi-factorized” TTI assumption, the inverted model (Figure 10) improves the positioning of the reflectors (Figure 12b) and reduces the residual moveout in CIGs (Figure 12a) and the VSP travelttime misfit.

However, the residual moveout in Figure 12a is not completely removed, mainly because the assumption about the anisotropy parameters does not conform to the actual \( \epsilon \)- and \( \delta \)-fields (Figures 7c and 7d). To allow for more realistic spatial variations, at the last stage of parameter updating we estimate the parameters \( \epsilon \) and \( \delta \) on the same grid as the one used for the velocity \( V_{P0} \). However, because the trade-offs between the parameters may cause large errors in \( \epsilon \) and \( \delta \) defined on small grids, the anisotropy parameters should be more tightly constrained, so the corresponding regularization coefficients should be larger than those for \( V_{P0} \).

After five more iterations with all three parameters updated on grids, the velocity \( V_{P0} \) above \( z = 7 \) km is relatively well-recovered with percentage errors in most areas smaller than 4% (Figure 11a). The spatial variations of \( \epsilon \) and \( \delta \) are partially resolved from the water bottom down to \( z = 5 \) km (Figure 11c and 11d). The coverage of VSP rays, however, becomes more sparse with depth. Also, the offset-to-depth ratio of P-wave reflections is insufficient to constrain the parameter \( \epsilon \) below 7 km, although the maximum offset reaches 10 km. Therefore, the accuracy in \( \epsilon \) and \( \delta \) decreases in the deep part of the model. The final inverted model (Figure 11) practically removes the residual moveout in CIGs (Figure 13a) (except for the locations close to the left and right edges due to poor ray coverage), and the reflections are better focused (Figure 13b), especially above \( z = 7 \) km.

An important parameter that influences the accuracy of the reconstructed velocity model is the symmetry-axis tilt \( \nu \), which is computed directly from the depth image. Poorly constrained TTI parameters in the deep part of the model yield a strongly distorted image, which produces large errors in the estimated values of \( \nu \). The obtained tilt field is used for the next iteration of MVA, which further distorts the other estimated TTI parameters. Therefore, without sufficient constraints from deep reflection events and VSP rays, the trade-offs between the tilt \( \nu \) and the other TTI parameters increase the uncertainty in velocity analysis at depth.
3.3 BP salt model

The last test is performed for another section of the BP TTI model that includes a salt dome (Figure 14). The strong reflections from the top of the salt dome and the flanks right beneath it are clearly imaged (Figure 15) by Kirchhoff depth migration, but the deeper segments of the flanks are blurred even when the correct model is used. The image quality can be improved with a wavefield-based imaging algorithm, such as reverse time migration (RTM).

The maximum offset (10 km) and the source and receiver intervals (50 m) are the same as in the previous test. The CIGs used for MVA are computed every 150 m from $x = 16$ km to 46 km. The dataset contains a vertical “well” at location $x_{\text{VSP}} = 29.9$ km to the left of the salt body (Figure 14). Two sets of 24 receivers (one between $z = 5275$ m and 5562.5 m and the other between $z = 8400$ m and 8687.5 m) were placed at even intervals in the well to record a walkaway VSP survey. The maximum offset for the VSP data is 10 km as well with a source interval of 50 m. The input data also include the check-shot traveltimes obtained every 50 m from $z = 1743.75$ m to 9093.75 m.

During the inversion, the water layer and salt body are kept isotropic with the velocities fixed at the correct values. Also, the actual positions of the top and flanks of the salt dome are assumed to be known, and the update is performed only for the sedimentary formations around the salt body. Since ray tracing becomes unstable in the presence of sharp velocity contrasts, we apply 2D smoothing to the velocity model to find the raypaths crossing the salt and then calculate the traveltimes and their derivatives in the original (unsmoothed) model.

Similar to the previous test, an initial isotropic model (Figure 6) is built from check-shot traveltimes. Because the TTI parameters are different on both sides of the salt body, the residual moveout in the CIGs (Figure 16a) is larger to the right of the salt (i.e., further away from the well). Due to the large velocity errors in the initial model, most reflectors are misplaced (Figure 16b).

After two iterations of velocity ($V_{P0}$) updating with fixed $\epsilon$ and $\delta$ and two more iterations with the “quasi-factorized” TTI model assumption (see above), the estimated parameter fields (Figure 17) produce relatively flat CIGs (Figure 19a) and an improved image (Figure 19b). For the VSP sources placed to the left of the well, the corresponding rays pass through the relatively simple sedimentary section. The VSP rays originated to the right of the well, however, cross the high-velocity salt body, and even small errors in the position of the salt boundary can cause large perturbations of the ray trajectories. Therefore, we assign smaller weights in the objective function to the VSP traveltimes for the sources to the right of the well. Hence, the anisotropic velocity field on the right side of the salt dome has to be determined mostly from the P-wave reflection data, which leads to larger errors in the TTI parameters.

To further reduce the residual moveout in CIGs and the VSP traveltime misfit, the anisotropy parameters are estimated on the same grid as that for $V_{P0}$. After three more iterations, the velocity (Figure 18a) above $z = 7$ km on the left side of the salt body is relatively well-resolved (errors in most areas do not exceed 3%); however, the errors in $V_{P0}$ on the right side are higher because of the limited constraints from VSP data, as described above. The spatial variations of $\epsilon$ and $\delta$ are partially recovered from the water bottom down to $z = 5$ km (Figure 18c and 18d). Because of the limited offset-to-depth ratio and poor coverage of VSP rays (especially for the right part of the model) at depth, the anisotropy parameters for grid points below 5 km could not be updated after the first iteration. Using the final model, the residual moveout in CIGs (Figure 20a) and the VSP traveltime misfit are largely reduced, which produces a more accurate image (Figure 20b).
Figure 7. Section of the BP TTI model that includes an anticline (the grid size is 6.25 m × 6.25 m). The top water layer is isotropic with velocity 1492 m/s. (a) The symmetry-direction velocity $V_{P0}$. The black line marks a vertical “well” at $x = 51.4$ km. (b) The symmetry-axis tilt $\nu$. The anisotropy parameters (c) $\epsilon$ and (d) $\delta$.

Figure 8. Depth image produced by prestack migration with the actual parameters from Figure 7. Imaging was performed using sources and receivers placed every 50 m.
Figure 9. (a) CIGs (displayed every 3 km from 41 km to 62 km) and (b) the migrated section computed with the initial model from Figure 5.
Figure 10. Anticline model from Figure 7 updated using the “quasi-factorized” TTI assumption. (a) The symmetry-direction velocity $V_{P0}$ estimated on a 200 m × 100 m grid. (b) The tilt $\nu$ obtained by setting the symmetry axis perpendicular to the reflectors. The inverted interval parameters (c) $\epsilon$ and (d) $\delta$, which are constant within each layer.

Figure 11. Inverted TTI parameters (a) $V_{P0}$, (c) $\epsilon$, and (d) $\delta$ after the final iteration of MVA. All three parameters are estimated on a 200 m × 100 m grid. (b) The symmetry-axis tilt $\nu$ computed from the depth image obtained before the final iteration.
Figure 12. (a) CIGs and (b) the migrated section obtained with the “quasi-factorized” TTI model from Figure 10.
Figure 13. (a) CIGs and (b) the migrated section computed with the final inverted model in Figure 11.
Figure 14. Section of the BP TTI model with a salt dome (the grid size is 6.25 m × 6.25 m). The top water layer and the salt body are isotropic with the P-wave velocity equal to 1492 m/s and 4350 m/s, respectively. (a) The symmetry-direction velocity \( V_{P0} \). The vertical “well” at \( x = 29.9 \) km is marked by a black line. (b) The tilt of the symmetry axis, which is set orthogonal to the interfaces. The anisotropy parameters (c) \( \epsilon \) and (d) \( \delta \).

Figure 15. Depth image produced with the actual parameters from Figure 14.
Figure 16. (a) CIGs (displayed every 3.25 km from 18 km to 44 km) and (b) the migrated section computed with the initial model in Figure 6.
Figure 17. Salt model from Figure 14 updated using the "quasi-factorized" TTI assumption. (a) The symmetry-direction velocity $V_{P0}$ estimated on a 200 m × 100 m grid. (b) The tilt $\nu$ obtained from the image. The inverted interval parameters (c) $\epsilon$ and (d) $\delta$, which are constant within each block.

Figure 18. Inverted TTI parameters (a) $V_{P0}$, (c) $\epsilon$, and (d) $\delta$ after the final iteration of joint tomography. All three parameters are estimated on a 200 m × 100 m grid. (b) The symmetry-axis tilt $\nu$ computed from the depth image obtained before the final iteration.
Figure 19. (a) CIGs and (b) the migrated section obtained with the “quasi-factorized” TTI model from Figure 17.
Figure 20. (a) CIGs and (b) the migrated section computed with the final inverted model in Figure 18.
### 4 CONCLUSIONS

Although most migration techniques have been extended to TTI media, accurate reconstruction of the anisotropic velocity field remains a difficult problem. Previously we developed an efficient 2D tomographic algorithm for heterogeneous TTI models, with the parameters $V_{P0}$, $\epsilon$, $\delta$, and the symmetry-axis tilt $\nu$ defined on a rectangular (in most cases square) grid. While $V_{P0}$, $\epsilon$, and $\delta$ are updated iteratively in the migrated domain, the tilt field is computed from the depth image by setting the symmetry axis perpendicular to the reflectors.

To resolve the TTI parameters in the presence of spatial velocity variations, here we combined reflection data with walkaway VSP and check-shot traveltimes. Our tomographic algorithm also incorporates useful geologic constraints by appropriately designed regularization. The regularization terms in the objective function allow for parameter variations across layers, but suppress them in the direction parallel to boundaries. In the iterative inversion, such structure-guided regularization also helps propagate along interfaces the most reliable updates corresponding to large derivatives in the Fréchet matrix (e.g., those in the cells crossed by dense VSP rays).

To improve the convergence of the algorithm, we propose a three-stage parameter-updating procedure. In the first several iterations, only the velocity $V_{P0}$ is updated on a grid, while the anisotropy parameters $\epsilon$ and $\delta$ are fixed at their initial values. This operation eliminates potentially large distortions in $\epsilon$ and $\delta$ caused by the parameter trade-offs. At the second stage of the inversion, $\epsilon$ and $\delta$ are taken constant in each layer and updated together with the grid-based velocity $V_{P0}$. Finally, all three TTI parameters are estimated simultaneously on the grid with the constraints provided by the regularization terms described above.

First, the algorithm was tested on a model that contains a bending TTI thrust sheet composed of several dipping blocks. Because of the relatively simple model geometry, we applied only the second stage of the updating procedure by estimating the velocity $V_{P0}$ on a grid, while keeping $\epsilon$ and $\delta$ constant in each layer. This “quasi-factorized” assumption proved sufficient to recover both $\epsilon$ and $\delta$ due to a wide range of available reflector dips. Without walkaway VSP traveltimes or strong regularization, however, the velocity $V_{P0}$ defined on a small grid could not be resolved just by flattening the CIGs for the available three reflectors.

Then the joint tomography of reflection and VSP data with structure-guided regularization was successfully applied to two sections of the BP TTI model that include an anticline and a salt dome. In both tests, a purely isotropic velocity field, which was obtained from check-shot traveltimes and extrapolated along the horizons, served as the initial model. With constraints from P-wave reflection and VSP data, the TTI parameters in the shallow part (above 5 km) of both sections are well-resolved. However, the errors in the anisotropy parameters $\epsilon$ and $\delta$ increase with depth due to the small offset-to-depth ratio and poor coverage of VSP rays. For the model with the salt dome, the anisotropic velocity field is recovered with higher accuracy to the left of the salt, where the inversion was tightly constrained by VSP data from a nearby well.

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### REFERENCES


