Image-domain wavefield tomography for acoustic VTI media

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## Seismic tomography methods

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Image-domain criteria

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<th>surface-offset CIGs</th>
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<tr>
<td>h</td>
<td>θ</td>
<td>λ_x</td>
</tr>
<tr>
<td>z</td>
<td>z</td>
<td>z</td>
</tr>
<tr>
<td>flatness</td>
<td>flatness</td>
<td>focusing at zero lag</td>
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Advantages

compared to ray-based tomography:

- based on wave-equation migration
- no reflector picking

compared to FWI:

- operates directly with reflections
- initial model can be inaccurate
- low sensitivity to source wavelet
Challenges

- artifacts in extended images
- low sensitivity to $\delta$
- trade-off between $V_{nmo}$ and $\eta$
Extended-domain objective function

\[ J = J_{DSO} + \alpha J_{PIP} \]

**DSO:** differential semblance optimization

**PIP:** partial image power
Idealized extended images

Inaccurate

Accurate
Extended images with artifacts

Inaccurate

Accurate
Preconditioning with approximation of $H^{-1}$

RTM

$m_0 = R^T d_r$
Preconditioning with approximation of $H^{-1}$

RTM

$$m_0 = R^T d_r$$

demigration and remigration

$$m_1 = R^T R m_0$$
Preconditioning with approximation of $H^{-1}$

\[ m_0 = R^T d_r \]

demigration and remigration

\[ m_1 = R^T R m_0 \]

solve

\[ \| m_0 - B m_1 \|^2 \approx 0 \]

$B$: nonstationary convolutional filter (Guitton, 2017)
Layered model
Extended space-lag CIG (understated $V_{nmo}$ and $\eta$)
Extended space-lag CIG

\[ \lambda_x \ (\text{km}) \]

\[ z \ (\text{km}) \]

**influence of**

\[ V_{nmo} \]
Extended space-lag CIG

$\lambda_x \text{ (km)}$

$z \text{ (km)}$

$\text{influence of } \eta$
Extended space-lag CIG

$\lambda_x (\text{km})$

$z (\text{km})$

RTM

aperture-truncation artifacts
Extended space-lag CIG

RTM

Pseudoinverse
Workflow

Reflection data (P-waves)

VTI model ($V_{nmo}$, $\eta$, $\delta$)

LSRTM gradient

matching filters

preconditioned LSRTM gradient

LSRTM image
Workflow

Reflection data (P-waves)

VTI model ($V_{nmo}$, $\eta$, $\delta$)

LSRTM gradient

preconditioned LSRTM gradient

matching filters

LSRTM image

$J_{DSO} + \alpha J_{IP}$
P-wave reflection moveout in VTI media

- $V_{nmo}$, $\eta$, and $\delta$

- low sensitivity to $\delta$

- trade-off between $V_{nmo}$ and $\eta$
Workflow

Reflection data (P-waves)

VTI model ($V_{nmo}$, $\eta$, $\delta$)

LSRTM gradient

matching filters

preconditioned LSRTM gradient

$J_{DSO} + \alpha J_{IP}$

$\Delta V_{nmo}, \Delta \eta$

LSRTM image
Workflow

- **Reflection data (P-waves)**
- **LSRTM gradient**
- **preconditioned LSRTM gradient**
- **matching filters**
- **VTI model** ($V_{nmo}$, $\eta$, $\delta$)
- **J_{DSO} + aJ_{IP}**
- **image-guided $\Delta V_{nmo}$, $\Delta \eta$**
Workflow

Reflection data (P-waves)

Borehole data (δ-profiles)

VTI model ($V_{nmo}, \eta, \delta$)

LSRTM gradient

matching filters

preconditioned LSRTM gradient

LSRTM image

$J_{DSO} + \alpha J_{IP}$

image-guided $\Delta V_{nmo}, \Delta \eta$

image-guided δ-field
Reflection data (P-waves) → LSRTM gradient → matching filters → preconditioned LSRTM gradient → LSRTM image → $J_{DSO} + \alpha J_{IP}$ → image-guided $\Delta V_{nmo}, \Delta \eta$ → updated VTI model ($V_{nmo}, \eta, \delta$)

Borehole data ($\delta$-profiles) → image-guided $\delta$-field

VTI model ($V_{nmo}, \eta, \delta$)
Stage 1 ($V_{nmo}$)

- Reflection data (P-waves)
- Borehole data ($\delta$-profiles)
- VTI model ($V_{nmo}$, $\eta$, $\delta$)
- LSRTM gradient
- matching filters
- preconditioned LSRTM gradient
- $J_{DSO} + \alpha J_{IP}$
- image-guided $\Delta V_{nmo}$
- updated VTI model ($V_{nmo}$, $\eta$, $\delta$)
- image-guided $\delta$-field
Stage 2 (η)

Reflection data (P-waves) → LSRTM gradient → matching filters → preconditioned LSRTM gradient

Borehole data (𝛿-profiles) → image-guided 𝛿-field

VTI model ($V_{nmo}$, $\eta$, $\delta$) → $J_{DSO} + \alpha J_{IP}$

image-guided $\Delta\eta$ → updated VTI model ($V_{nmo}$, $\eta$, $\delta$)
Stage 3 ($V_{nmo, \eta}$)

**Reflection data** (P-waves)

**Borehole data** ($\delta$-profiles)

- VTI model ($V_{nmo, \eta, \delta}$)
  - LSRTM gradient
    - matching filters
  - preconditioned LSRTM gradient
    - $J_{DSO} + \alpha J_{IP}$
      - image-guided $\Delta V_{nmo, \Delta \eta}$
      - updated VTI model ($V_{nmo, \eta, \delta}$)
  - image-guided $\delta$-field
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<td>artifacts in extended images:</td>
<td>preconditioned LSRTM</td>
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<td>low sensitivity to $\delta$:</td>
<td>image-guided interpolation</td>
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<tr>
<td>trade-off between $V_{nmo}$ and $\eta$:</td>
<td>multistage algorithm</td>
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Marmousi-II model ($V_{nmo}$)
Anisotropy parameters
Initial $V_{nmo}$ (initial $\eta=\delta=0$)
RTM with initial model
LSRTM with initial model
Parameter $\delta$ from interpolation
Stage 1: Update $V_{nmo}$
$V_{nmo}$ after stage 1
$V_{nmo}$ before stage 1
Extended LSRTM CIGs at $x = 3$ km

Before and after comparisons.
Extended LSRTM CIGs at $x = 5$ km

before

after
Stage 2: Update $\eta$
Parameter $\eta$ after stage 2 (initial $\eta = 0$)
Stage 3: Update $V_{nmo}$ and $\eta$
\( V_{nmo} \) after stage 3
$V_{nmo}$ before stage 3
Parameter $\eta$ after stage 3
Parameter $\eta$ before stage 3
Parameter $\delta$ after stage 3
LSRTM with inverted model
LSRTM with initial model
Gulf of Mexico line (courtesy of Shell)
Initial $\delta = \varepsilon$ ($\eta = 0$)
Data preprocessing

- projection onto line
- debubbling
- P-Z summation
- removal of direct arrival
- smoothed-envelope normalization
Preprocessed data
LSRTM objective function

\[ J = \frac{1}{2} \left\| W(d^{\text{obs}} - d^{\text{cal}}) \right\|^2 \]
RTM with initial model
LSRTM with initial model
Extended LSRTM CIGs with initial model

A

B

C
Preconditioned DSO $\eta$-gradient
Updated $\eta$-field
LSRTM with updated $\eta$
LSRTM with initial model
Initial $\delta = \varepsilon$ ($\eta = 0$)
Challenges

- Artifacts in extended images: preconditioned LSRTM
- Low sensitivity to $\delta$: image-guided interpolation
- Trade-off between $V_{nmo}$ and $\eta$: three-stage algorithm
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Workflow

- Reflection data (P-waves)
- Borehole data (δ-profiles)

- VTI model \((V_{nmo}, \eta, \delta)\)
- LSRTM gradient
- matching filters
- preconditioned LSRTM gradient

- \(J_{DSO} + \alpha J_{IP}\)
- Image-guided \(\Delta V_{nmo}, \Delta \eta\)
- updated VTI model \((V_{nmo}, \eta, \delta)\)

- image-guided δ-field