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# Extracting the wave-function in quantum mechanics from intensity measurements

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**ABSTRACT**
Existing methods for Green's function extraction give the Green's function that accounts for wave propagation between two points from the correlation of field fluctuations recorded at those points. In this work it is shown that the Green's function of the Schrödinger equation can be retrieved from intensity measurements taken from three experiments where two time-harmonic sources first operate separately, and then simultaneously. This makes it possible to infer the phase and amplitude of the wave-function from intensity measurements over a closed surface surrounding both sources.

**Key words:** seismic interferometry, Green's function extraction, quantum mechanics

## 1 INTRODUCTION

Extracting the Green's function from field fluctuations is a technique that has gone through a rapid growth (Larose *et al.*, 2006; Curtis *et al.*, 2006; Wapenaar *et al.*, 2008; Schuster, 2009). The principle of Green's function extraction has been formulated for the Schrödinger equation (Snieder *et al.*, 2007) and thus is, in principle, applicable to quantum mechanics. In that application one can retrieve the wave function by cross correlating recorded field fluctuations excited by uncorrelated sources on a closed surface surrounding the receivers. This approach is, however, not practical since it presumes that one can measure the wave-function which is in general not accessible to direct observation.

In this work we present a different method that allows extraction of the wave-function by considering a sequence of three experiments with time-harmonic sources at two locations $\mathbf{r}_A$ and $\mathbf{r}_B$. In these experiments one measures first the probability density flux through a closed surface surrounding the sources when each of the sources is used separately, and next measures the probability density flux when both sources are used simultaneously. We show that these measurements of the integrated probability flux can be used to determine both the amplitude and phase of the Green's function $G(\mathbf{r}_A, \mathbf{r}_B)$. Our method extracts the Green's function between two source locations, as in ref. (Curtis *et al.*, 2009), the key difference being that we show that measurements of the integrated intensity are sufficient to extract the full Green's function, including the phase.

Existing methods for phase retrieval, e.g. (Bunge *et al.*, 1974; Toda, 1992; Orlowski & Paul, 1994, Spence *et al.*, 2002), infer the phase of the wave function from intensity measurements taken at the location where the phase is retrieved. In the method proposed here, the amplitude and phase of the Green's function is inferred from integrated intensity measurements at a closed surface away from these sources. In holography, phase information is restored from the interference of a reference wave with waves reflected off an object (Lauterborn *et al.*, 1995). In this work the phase information is retrieved from the interference of waves that are excited by different time-harmonic sources.

## 2 RETRIEVING THE GREEN'S FUNCTION FROM INTENSITIES

Consider the Schrödinger equation in the frequency domain

$$\nabla^2\psi(\mathbf{r}) + \left(k^2 - \frac{2m}{\hbar^2}V(\mathbf{r})\right)\psi(\mathbf{r}) = 4\pi q(\mathbf{r}) \;, \qquad (1)$$

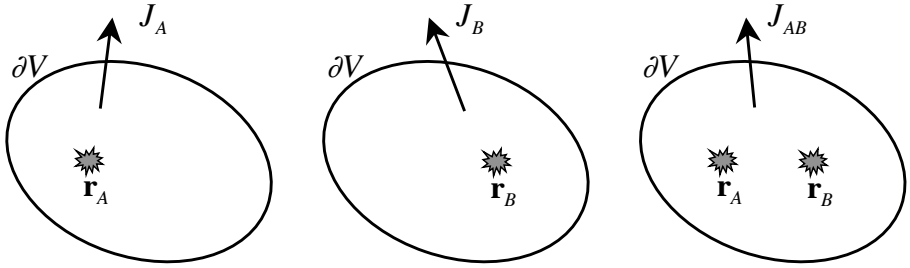


**Figure 1.** Three source configurations and their probability density currents. The currents in the three configurations correspond to fluxes $\Phi_A$, $\Phi_B$, and $\Phi_{AB}$, respectively.

where $\hbar$ is Planck's constant divided by $2\pi$, $m$ the mass, $\omega$ the angular frequency, $k = \sqrt{2m\omega/\hbar}$ the wave number, $V(\mathbf{r})$ a real, but otherwise arbitrary, potential, and $q(\mathbf{r})$ an inhomogeneous source term. The theory presented here is valid for time-harmonic fields with time dependence $e^{-i\omega t}$.

We next derive an expression for the probability density current by considering $\psi^*$(E1)-$\psi$(E1)$^*$, where the asterisk denotes complex conjugation and (E1) stands for equation (1). Integrating the result over an arbitrary volume $V$, with boundary $\partial V$, gives

$$\int_V \left(\psi^* \nabla^2 \psi - \psi \nabla^2 \psi^*\right) dV = 4\pi \int_V \left(q\psi^* - q^* \psi\right) dV \ . \tag{2}$$

Applying Gauss' theorem and using the following definition of the probability density flux (Merzbacher, 1970)

$$\Phi = (\hbar/2mi) \oint_{\partial V} \left(\psi^* \nabla \psi - \psi \nabla \psi^*\right) \cdot d\mathbf{S} \ , \tag{3}$$

reduces expression (2) to

$$-\frac{im}{2\pi\hbar}\Phi = \int_V \left\{q^* \psi - q\psi^*\right\} dV \ . \tag{4}$$

We consider the three experiments shown in figure 1. First a time-harmonic source is present at location $\mathbf{r}_A$. Then the experiment is repeated with a source only at location $\mathbf{r}_B$, and then two time-harmonic sources are used simultaneously at locations $\mathbf{r}_A$ and $\mathbf{r}_B$. For each source configuration the flux through $\partial V$ is denoted by $\Phi_A$, $\Phi_B$, and $\Phi_{AB}$, respectively. In the experiment in the left panel of figure 1 the source is $q(\mathbf{r}) = \delta(\mathbf{r} - \mathbf{r}_A)$, the response to this point source is given by the Green's function $G(\mathbf{r}, \mathbf{r}_A)$, hence equation (4) reduces to

$$-\frac{im}{2\pi\hbar}\Phi_A = G(\mathbf{r}_A, \mathbf{r}_A) - G^*(\mathbf{r}_A, \mathbf{r}_A) \ . \tag{5}$$

The right hand side of this equation is equal to $2iIm(G(\mathbf{r}_A, \mathbf{r}_A))$, with $Im$ denoting the imaginary part. Since the imaginary part of the Green's function satisfies a homogeneous equation, it is finite at the source (Oristaglio, 1989; Snieder *et al.*, 2009), and the right hand side of equation (5) therefore is finite. The experiment in the middle panel of figure 1 gives the same result for the source at $\mathbf{r}_B$

$$-\frac{im}{2\pi\hbar}\Phi_B = G(\mathbf{r}_B, \mathbf{r}_B) - G^*(\mathbf{r}_B, \mathbf{r}_B) \ . \tag{6}$$

In the experiment in the right panel of figure 1 the excitation is given by $q(\mathbf{r}) = \delta(\mathbf{r} - \mathbf{r}_A) + \delta(\mathbf{r} - \mathbf{r}_B)$, and the field therefore is given by $G(\mathbf{r}, \mathbf{r}_A) + G(\mathbf{r}, \mathbf{r}_B)$. The first term in the right hand side of equation (4) now is equal to $\int_V q^* \psi dV = \int_V \left\{(\delta(\mathbf{r} - \mathbf{r}_A) + \delta(\mathbf{r} - \mathbf{r}_B))\,(G(\mathbf{r}, \mathbf{r}_A) + G(\mathbf{r}, \mathbf{r}_B))\right\} = G(\mathbf{r}_A, \mathbf{r}_A) + 2G(\mathbf{r}_A, \mathbf{r}_B) + G(\mathbf{r}_B, \mathbf{r}_B)$, where the properties of the delta function and reciprocity $G(\mathbf{r}_A, \mathbf{r}_B) = G(\mathbf{r}_B, \mathbf{r}_A)$ are used in the last identity. Using this in equation (4) yields

$$-\frac{im}{2\pi\hbar}\Phi_{AB} = G(\mathbf{r}_A, \mathbf{r}_A) + 2G(\mathbf{r}_A, \mathbf{r}_B) + G(\mathbf{r}_B, \mathbf{r}_B)$$
$$-G^*(\mathbf{r}_A, \mathbf{r}_A) - 2G^*(\mathbf{r}_A, \mathbf{r}_B) - G^*(\mathbf{r}_B, \mathbf{r}_B) \ . \tag{7}$$

Subtracting equations (5) and (6) from equation (7) gives

$$G(\mathbf{r}_A, \mathbf{r}_B) - G^*(\mathbf{r}_A, \mathbf{r}_B) = \frac{im}{4\pi\hbar}\left(\Phi_A + \Phi_B - \Phi_{AB}\right) \ . \tag{8}$$

The left hand side gives the imaginary part of $G(\mathbf{r}_A, \mathbf{r}_B)$. One might think that this is not enough information to recover the full Green's function, but after a Fourier transform to the time domain the left hand side corresponds to $G(\mathbf{r}_A, \mathbf{r}_B, t) - G(\mathbf{r}_A, \mathbf{r}_B, -t)$. Since the Green's function is causal, $G(\mathbf{r}_A, \mathbf{r}_B, t)$ is only nonzero for $t > 0$ and $G(\mathbf{r}_A, \mathbf{r}_B, -t)$ is only nonzero for $t < 0$. By parsing these contributions the full Green's functions can be determined. The right hand side of equation (8) depends on the measured fluxes. These are real numbers that are accessible to observation. This means that the right hand side of expression (8), which can be measured, can be used with the sequence of three experiments of figure 1 to obtain the full Green's function, including its phase.

## 3 EXAMPLE, TWO-SLIT EXPERIMENT

We illustrate the theory with a two-slit experiment where a plane wave is incident on a screen with two

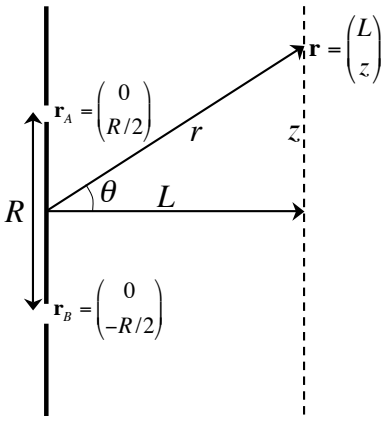


**Figure 2.** Definition of geometric variables for the two-slit experiment.

slits separated by a distance $R$ (figure 2). We assume that the slits are very long in the $y$-direction, so that the problem can be treated as a two-dimensional problem in the $(x, z)$-plane. The plane wave incident on the slits effectively act as point sources in the $(x, z)$-plane at locations $\mathbf{r}_A$ and $\mathbf{r}_B$, respectively. The intensity of the waves is measured at a recording screen indicated by the dashed line. The surface $\partial V$ consists of the screen on the left (solid line of figure 2), the recording screen (dashed line), and two segments joining the two screens. The latter segments do not contribute when taken at infinity. Since the particles do not move through the screen with the slits, the flux through this screen vanishes as well. This reduces the surface integral to an integration over the recording screen.

We consider the case where the potential to the right of the screen vanishes (free particle), hence the Green's function solution to equation (1) is given by (Arfken & Weber, 2001)

$$G(\mathbf{r}, \mathbf{r}_0) = -i\pi H_0^{(1)}(k|\mathbf{r} - \mathbf{r}_0|) \; , \tag{9}$$

where $H_0^{(1)}$ is the zeroth order Hankel function of the first kind. The distance between a point $\mathbf{r}$ on the recording screen and the slits is given by $|\mathbf{r} - \mathbf{r}_{A,B}| = \sqrt{L^2 + (z \mp R/2)^2}$, where the upper sign is for the slit at $\mathbf{r}_A$ and the lower sign for the one at $\mathbf{r}_B$. We assume in the following that the distance $R$ between the slits is much smaller than the horizontal distance $L$ to the recording screen and only retain terms to first order in $R/L$ or $R/r$. In this approximation

$$|\mathbf{r} - \mathbf{r}_{A,B}| = r \mp zR/2r \; . \tag{10}$$

When the distance $L$ is much larger than a wavelength, we can use the far field approximation to the Hankel function (equation (11.131) of ref. (Arfken & Weber, 2001)), and $G(\mathbf{r}, \mathbf{r}_0) = -\sqrt{2\pi/(k|\mathbf{r} - \mathbf{r}_0|)}\exp(i(k|\mathbf{r} - \mathbf{r}_0| + \pi/4))$. We use the approximation (10) in the exponent and replace the distance $|\mathbf{r} - \mathbf{r}_0|$ in the denominator by $r$, so that

$$G(\mathbf{r}, \mathbf{r}_{A,B}) = -\sqrt{\frac{2\pi}{kr}} e^{i(kr \mp kzR/2r + \pi/4)} \; . \tag{11}$$

The flux depends on the derivative of the Green's function perpendicular to the screen. Retaining terms to first order in $R/r$ only in the exponent (consistent with approximations made earlier) gives

$$\frac{\partial G(\mathbf{r}, \mathbf{r}_{A,B})}{\partial x} = -\frac{ikL}{r}\sqrt{\frac{2\pi}{kr}} e^{i(kr \mp kzR/2r + \pi/4)} \; . \tag{12}$$

When only the slit at $\mathbf{r}_A$ is open, it follows from expressions (3), (11), and (12) that the intensity flux through the recording screen is given by

$$\Phi_A = \frac{2\pi\hbar L}{m} \int_{-\infty}^{\infty} \frac{1}{r^2} dz \; . \tag{13}$$

According to figure 2, $r = L/\cos\theta$ and $z = L\tan\theta$, hence $\int_{-\infty}^{\infty} 1/r^2 dz = L^{-1}\int_{-\pi/2}^{\pi/2} d\theta = \pi/L$ and

$$\Phi_A = \Phi_B = \frac{2\pi^2\hbar}{m} \; . \tag{14}$$

(Since the slits are identical, we have used that $\Phi_A = \Phi_B$.)

When both slits are open, the response is given by $G(\mathbf{r}, \mathbf{r}_A) + G(\mathbf{r}, \mathbf{r}_B)$, and the far field expressions (11) and (12) give

$$\Phi_{AB} = \frac{4\pi\hbar L}{m} \int_{-\infty}^{\infty} \left\{ \frac{1}{r^2} + \frac{\cos(kzR/r)}{r^2} \right\} dz \; . \tag{15}$$

Expressing $z$ and $r$ as defined in figure 2 into the angle $\theta$, reduces the last integral to

$$\begin{aligned} \int_{-\infty}^{\infty} \frac{\cos(kzR/r)}{r^2} dz \quad &= \frac{1}{L}\int_{-\pi/2}^{\pi/2} \cos(kR\sin\theta) d\theta \\ &= \frac{\pi}{L} J_0(kR) \; , \end{aligned} \tag{16}$$

where we used the integral representation given in expression (11.30) of ref. (Arfken & Weber, 2001) for the zeroth order Bessel function $J_0$ in the last identity. Expression (15) thus reduces to

$$\Phi_{AB} = \frac{4\pi^2\hbar}{m} \left\{ 1 + J_0(kR) \right\} \; . \tag{17}$$

Using this, and expression (14), gives

$$\begin{aligned} \frac{im}{4\pi\hbar} \left( \Phi_A + \Phi_B - \Phi_{AB} \right) &= -i\pi J_0(kR) \\ &= -i\pi Im\left( iH_0^{(1)}(kR) \right) \; , \end{aligned} \tag{18}$$

where we used the relation $H_0^{(1)}(x) = J_0(x) + iN_0(x)$ (expression (11.85) of ref. (Arfken & Weber, 2001)) in the last identity. From expression (9), $Im\left(iH_0^{(1)}(kR)\right) = (-1/\pi) Im\left(G(R)\right) = (-1/2\pi i)(G(R) - G^*(R))$, and using that $G(R) = G(\mathbf{r}_A, \mathbf{r}_B)$ reduces equation (18) to

$$\frac{im}{4\pi\hbar} \left( \Phi_A + \Phi_B - \Phi_{AB} \right) = \frac{1}{2} G(\mathbf{r}_A, \mathbf{r}_B) - \frac{1}{2} G^*(\mathbf{r}_A, \mathbf{r}_B) \; . \tag{19}$$

Apart from a factor 1/2, the expression above is identical to the general equation (8). The factor 1/2 is due the fact that the delta functions used to account for the diffraction through the slits radiate in both directions, while the slits only radiate energy to the right. Note that expression (19) is based on the far field approximations (11)-(13), yet it gives the exact Green's function, even when the slits are in each other's near field ($kR = O(1)$).

The dual slit example shows explicitly that the Green's function that accounts for wave propagation between two slits is related to the differences of the particle flux measured at a screen when each slit is open at a time and when both slits are open. In the latter case the waves radiated by the slits interfere, and it is the imprint of this interference on the intensity flux that leads to the Green's function that accounts for the propagation between the slits.

## 4 ACOUSTICS

The treatment of section 2 for the Schrödinger equation can be extended to acoustic waves. The equation of motion and the constitutive equation for acoustic waves are given by

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla p \quad , \quad \kappa \frac{\partial p}{\partial t} = -(\nabla \cdot \mathbf{v}) + \frac{\partial q}{\partial t} \; , \tag{20}$$

where $\rho$ is the mass-density, $\mathbf{v}$ the particle velocity, $\kappa$ the compressibility, $p$ the pressure, and $q$ the injection source. Both $\kappa$ and $\rho$ can be arbitrary functions of space. Using the Fourier convention $f(t) = \int F(\omega) \exp(-i\omega t) d\omega$ these equations correspond in the frequency domain to

$$\nabla p - i\omega\rho \mathbf{v} = 0 \; , \tag{21}$$

and

$$(\nabla \cdot \mathbf{v}) - i\omega\kappa p = -i\omega q \; . \tag{22}$$

We derive an expression for the energy current by forming the combination $\mathbf{v}^*(\text{E21}) + p(\text{E22})^* + \mathbf{v}(\text{E21})^* + p^*(\text{E22})$, where $(\text{E22})^*$ denotes, for example, the complex conjugate of equation (22). Forming this combination, integrating over volume and applying Gauss' law gives

$$-\frac{4i}{\omega} \oint_{\partial V} \mathbf{J} \cdot d\mathbf{S} = \int_V \{q^* p - q p^*\} \, dV \; , \tag{23}$$

where $J$ is the energy current density (Morse & Ingard, 1968)

$$\mathbf{J} = \frac{1}{4} \left( p\mathbf{v}^* + p^* \mathbf{v} \right) \; . \tag{24}$$

Associated with this energy density current is a total energy flux $\Phi = \oint_{\partial V} \mathbf{J} \cdot d\mathbf{S}$ through $\partial V$, and equation (23) reduces to the counterpart of expression (4)

$$-\frac{4i}{\omega} \Phi = \int_V \{q^* p - q p^*\} \, dV \; , \tag{25}$$

The same three experiments shown in figure 1 can be applied to acoustic waves, and using the reasoning that led to equation (8) gives for acoustic waves

$$G(\mathbf{r}_A, \mathbf{r}_B) - G^*(\mathbf{r}_A, \mathbf{r}_B) = \frac{2i}{\omega} \left( \Phi_A + \Phi_B - \Phi_{AB} \right) \; . \tag{26}$$

This means that for acoustic wave the Green's function that accounts for wave propagation between $\mathbf{r}_A$ and $\mathbf{r}_B$ can be constructed by measuring the acoustic energy fluxes through $\partial V$ for the three source configurations of figure 1. Just as in the quantum mechanical case the full Green's function, including the phase, can be found by measuring fluxes for three different experiments.

## 5 COMPLICATIONS

In practice the amplitude and phase of the sources at $\mathbf{r}_A$ and $\mathbf{r}_B$ may differ. Consider the case where the source at $\mathbf{r}_A$ has a phase difference $\varphi$ and relative amplitude $A$ compared with the source at $\mathbf{r}_B$. In that case the source at $\mathbf{r}_A$ is given by $q(r) = A \exp(i\varphi) \delta(\mathbf{r} - \mathbf{r}_A)$, and the associated field is $A \exp(i\varphi) G(\mathbf{r}, \mathbf{r}_A)$. The source at $\mathbf{r}_B$ and its related field are unchanged. Repeating the derivation leading to expression (8) gives in this case

$$\begin{aligned} &\{G(\mathbf{r}_A, \mathbf{r}_B) - G^*(\mathbf{r}_A, \mathbf{r}_B)\} \, A \cos\varphi \\ &\qquad = \frac{im}{4\pi\hbar} \left( \Phi_A + \Phi_B - \Phi_{AB} \right) \; . \end{aligned} \tag{27}$$

A phase shift and amplitude difference between the sources at $\mathbf{r}_A$ and $\mathbf{r}_B$ thus only gives an overall amplitude change. Note that when the sources are 90 degrees out off phase ($\varphi = \pm\pi/2$) the left hand side of expression (27) vanishes, in that case the Green's function retrieval breaks down.

Note that it is not necessary to know the phase of the sources, and that we did not assume that the phase of the sources in the three experiments of figure 1 is identical. The method proposed here only requires that the sources have a constant phase difference $\varphi$. This means that when one averages over a long time or over an ensemble, as one does in practice, the sources need not be coherent in time, as long as their phase *difference* is constant. We illustrate in section 3 how this can be achieved using a screen with holes.

In experiments the source at $\mathbf{r}_A$ may not be a point source, but a finite source distribution $q(\mathbf{r}) = S(\mathbf{r}_A - \mathbf{r})$ centered at $\mathbf{r}_A$. In that case $\psi(\mathbf{r}) = \int G(\mathbf{r}, \mathbf{r}') S(\mathbf{r}_A - \mathbf{r}') dV'$, the right hand side of equation (4) contains $\int q^*(\mathbf{r}) \psi(\mathbf{r}) dV = \int\int S^*(\mathbf{r}_A - \mathbf{r}) G(\mathbf{r}, \mathbf{r}') S(\mathbf{r}_A - \mathbf{r}') dV dV'$. Suppose that the source at $\mathbf{r}_B$ is given by the same distribution $S(\mathbf{r}_B - \mathbf{r})$, but now centered at $\mathbf{r}_B$. Repeating the derivation of section 2 then gives

$$\begin{aligned} &\int\int S^*(\mathbf{r}_A - \mathbf{r}) \left( G(\mathbf{r}, \mathbf{r}') - G^*(\mathbf{r}, \mathbf{r}') \right) S(\mathbf{r}_B - \mathbf{r}') dV dV' \\ &\qquad = \frac{im}{4\pi\hbar} \left( \Phi_A^S + \Phi_B^S - \Phi_{AB}^S \right) \; , \end{aligned} \tag{28}$$

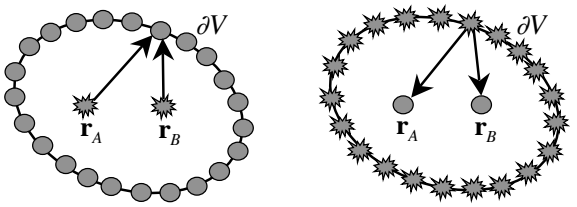


**Figure 3.** Comparison of the traditional method for Green's function extraction (Wapenaar et al., 2005) from random sources (right panel), with the method for Green's function extraction proposed in this work (left panel). Sources are denoted by stars, receivers by circles.

where $\Phi^S$ denotes the flux generated by the sources $S$. In this case a double convolution of the Green's function with the source function is obtained. By deconvolution one can, in principle, obtain $G - G^*$. In practice this may not be possible for all frequencies, in that case one can only obtain a band-limited version of $G - G^*$. Note that equation (28) only holds when the sources centered around $\mathbf{r}_A$ and $\mathbf{r}_B$ are identical, as is the case when using identical holes or slits in a screen.

## 6 DISCUSSION

The theory here provides a method to obtain the phase and amplitude of the Green's function for the Schrödinger equation from measurements of the integrated intensity flux through a closed surface surrounding two sources. Note that the theory holds for an arbitrary real potential $V(\mathbf{r})$. The example of the two-slit experiment of section 3 shows that two slits, or pin-holes, in a screen can be used as sources for the method described here. As shown in equation (27) the used sources need not be in phase, and their amplitudes may be different.

The method for Green's function extraction is based on the subtraction of expectation values for an arbitrary operator $O$

$$\langle \psi_A + \psi_B | O | \psi_A + \psi_B \rangle - \langle \psi_A | O | \psi_A \rangle - \langle \psi_B | O | \psi_B \rangle = \\ \langle \psi_A | O | \psi_B \rangle + \langle \psi_B | O | \psi_A \rangle \ , \tag{29}$$

which follows from the bilinear properties of the expectation value. Note that the right hand side only contains cross-terms of the two states $\psi_A$ and $\psi_B$, hence the subtraction selects *interference terms* between the two states. Similarly, the right hand side of (8) gives cross-terms proportional to $C_{AB} - C^*_{AB}$ with

$$C_{AB} = \oint_{\partial V} (G(\mathbf{r}, \mathbf{r}_A) \nabla G^*(\mathbf{r}, \mathbf{r}_B) \\ - G^*(\mathbf{r}, \mathbf{r}_B) \nabla G(\mathbf{r}, \mathbf{r}_A)) \cdot d\mathbf{S} \ . \tag{30}$$

This integral has the same form as the integral used in Green's function extraction for acoustic waves (expression (9) of ref. (Wapenaar *et al.*, 2005)), except that in equation (30) the Green's function has arguments $G(\mathbf{r}, \mathbf{r}_{A,B})$ whereas in traditional Green's function extraction the integrand depends on $G(\mathbf{r}_{A,B}, \mathbf{r})$ (Wapenaar *et al.*, 2005). The integral (30) corresponds to the situation shown in the left panel of figure 3 where the flux of the field excited by sources at $\mathbf{r}_A$ and $\mathbf{r}_B$ is measured at locations $\mathbf{r}$ at $\partial V$. In Green's function extraction, as shown in the right panel of figure 3, one cross-correlates the fields measured at locations $\mathbf{r}_A$ and $\mathbf{r}_B$ that are excited by uncorrelated sources at locations $\mathbf{r}$ on the surface (Wapenaar *et al.*, 2005). Because of reciprocity these two cases are identical. It was recognized earlier (Curtis *et al.*, 2009) that cross-correlation methods can yield the waves that propagate between sources. In this work we generalize this principle to obtain the Green's function from measurements of the integrated intensity flux.

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