GEOPHYSICS, VOL. 68, NO. 5 (SEPTEMBER-OCTOBER 2003); P. 1600–1610, 8 FIGS.
10.1190/1.1620634

Quartic moveout coefficient: 3D description and application to tilted TI media

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ABSTRACT

Nonhyperbolic (long-spread) moveout provides essential information for a number of seismic inversion/processing applications, particularly for parameter estimation in anisotropic media. Here, we present an analytic expression for the quartic moveout coefficient A_4 that controls the magnitude of nonhyperbolic moveout of pure (nonconverted) modes. Our result takes into account reflection-point dispersal on irregular interfaces and is valid for arbitrarily anisotropic, heterogeneous media. All quantities needed to compute A_4 can be evaluated during the tracing of the zero-offset ray, so long-spread moveout can be modeled without time-consuming multioffset, multiazimuth ray tracing.

The general equation for the quartic coefficient is then used to study azimuthally varying nonhyperbolic moveout of P-waves in a dipping transversely isotropic (TI) layer with an arbitrary tilt ν of the symmetry axis. Assuming that the symmetry axis is confined to the dip plane, we employed the weak-anisotropy approximation to analyze the dependence of A_4 on the anisotropic parameters. The linearized expression for A_4 is proportional to the anellipticity coefficient $\eta \approx \epsilon - \delta$ and does not depend on the individual values of the Thomsen parameters. Typically, the magnitude of nonhyperbolic moveout in tilted TI media above a dipping reflector is highest near the reflector strike, whereas deviations from hyperbolic moveout on the dip line are substantial only for mild dips.

The azimuthal variation of the quartic coefficient is governed by the tilt ν and reflector dip ϕ and has a much more complicated character than the NMO–velocity ellipse. For example, if the symmetry axis is vertical (VTI media, $\nu = 0$) and the dip $\phi > 30^\circ$, A_4 goes to zero on two lines with different azimuths where it changes sign. If the symmetry axis is orthogonal to the reflector (this model is typical for thrust-and-fold belts), the strike-line quartic coefficient is defined by the well-known expression for a horizontal VTI layer (i.e., it is independent of dip), while the dip-line A_4 is proportional to $\cos^4 \phi$ and rapidly decreases with dip. The high sensitivity of the quartic moveout coefficient to the parameter η and the tilt of the symmetry axis can be exploited in the inversion of wide-azimuth, long-spread P-wave data for the parameters of TI media.

INTRODUCTION

In conventional seismic data processing, reflection moveout of pure (nonconverted) modes is typically assumed to be hyperbolic, at least for spread lengths not exceeding reflector depth. However, the presence of heterogeneity (either lateral or vertical) or anisotropy causes deviations from hyperbolic moveout which sometimes cannot be ignored, even for offsets-to-depth ratios close to unity (e.g., Al-Dajani and Tsvankin, 1998). Insufficient understanding of nonhyperbolic moveout and practical difficulties in working with long-spread data often force seismic processors to mute out the nonhyperbolic portion of the moveout curve. Long-spread moveout, however, has proved useful in a number of applications, such as anisotropic parameter estimation, suppression of multiples, and large-angle amplitude variation with offset (AVO) analysis.

A detailed overview of existing results on nonhyperbolic moveout analysis in anisotropic media can be found in Tsvankin (2001). Most earlier work on the contribution of anisotropy to long-spread moveout (e.g., Hake et al., 1984; Byun and Corrigan, 1990; Muir et al., 1993) is restricted to transversely isotropic (TI) models with a vertical symmetry axis (VTI). Tsvankin and Thomsen (1994) developed a general nonhyperbolic moveout equation based on the normal-moveout

Manuscript received by the Editor July 9, 2002; revised manuscript received March 5, 2003.
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(NMO) velocity V_{nmo} and the quartic moveout coefficient A_4 of the $t^2(x^2)$–function. In contrast to the conventional Taylor series, the Tsvankin-Thomsen equation converges at offsets approaching infinity, which ensures its high accuracy in the intermediate offset range (i.e., for offsets two to three times the reflector depth) important in reflection seismology. A particularly convenient form of this equation for P-waves in VTI media was suggested by Alkhalifah and Tsvankin (1995) (also see Grechka and Tsvankin, 1998a), who showed that P-wave reflection moveout, as well as other time-domain signatures, is controlled by just the NMO velocity and the anellipticity coefficient η defined as $\eta \equiv (\epsilon - \delta)/(1 + 2\delta)$, where ϵ and δ are Thomsen's (1986) parameters. The equation of Alkhalifah and Tsvankin (1995) has been widely used for estimating the parameter η from P-wave long-spread traveltimes and building vertically heterogeneous VTI models in the time domain (e.g., Alkhalifah, 1997; Toldi et al., 1999).

The behavior of nonhyperbolic moveout becomes much more complicated if the medium is azimuthally anisotropic. Al-Dajani and Tsvankin (1998) derived the quartic coefficient A_4 for transversely isotropic media with a horizontal symmetry axis (HTI) and extended the Tsvankin-Thomsen equation to layer-cake HTI media. A different method based on spherical harmonics was employed by Sayers and Ebrom (1997) to describe long-spread P-wave moveout in a horizontal azimuthally anisotropic layer. It should be emphasized that all the papers listed above treat models with a horizontal symmetry plane, in which the derivation of the coefficient A_4 for pure modes is simplified by the absence of reflection-point dispersal on common-midpoint (CMP) gathers. Fomel and Grechka (2001) developed a more general approach to the analytic description of nonhyperbolic moveout that accounts for reflection-point dispersal at dipping or curved interfaces. They also applied their methodology to P-wave moveout in heterogeneous VTI media.

Here, we introduce a general 3D expression for the quartic moveout coefficient and use it to describe nonhyperbolic moveout of P-waves for TI media with an arbitrary tilt of the symmetry axis. Models with the symmetry axis tilted away from the vertical [tilted TI (TTI) media] are typical for sediments near the flanks of salt domes and fold-and-thrust belts such as the Canadian Foothills (Tsvankin, 1997, 2001; Isaac and Lawton, 1999). A symmetry axis tilted at an oblique angle creates an azimuthally anisotropic model without a horizontal symmetry plane, where nonhyperbolic moveout is influenced by reflection-point dispersal. We present analytic expressions for the quartic moveout term for both horizontal and dipping reflectors and study the azimuthal dependence of nonhyperbolic moveout as a function of reflector dip and symmetry-axis orientation.

ANALYTIC DESCRIPTION OF NONHYPERBOLIC MOVEOUT

Reflection moveout of pure (nonconverted) modes is conventionally approximated by a Taylor series expansion of the squared traveltime t^2, which is often truncated after the quartic term (Taner and Koehler, 1969):

$$t^2 = A_0 + A_2 X^2 + A_4 X^4, \tag{1}$$

where X is the source-receiver offset and

$$A_0 = t_0^2, \quad A_2 = \left.\frac{d(t^2)}{d(X^2)}\right|_{X=0},$$

$$A_4 = \frac{1}{2}\frac{d}{d(X^2)}\left[\frac{d(t^2)}{d(X^2)}\right]\Bigg|_{X=0}. \tag{2}$$

Here, $t_0 = t(0)$ is the zero-offset traveltime, and A_2 is related to the NMO velocity as $A_2 = V_{nmo}^{-2}$. The first two terms in equation (1) represent the hyperbolic part of the moveout curve, whereas the quartic coefficient, A_4, describes nonhyperbolic moveout.

Although the three-term series given by equation (1) provides a better approximation for long-spread moveout than the conventional hyperbolic equation based on just V_{nmo}, it loses accuracy for offsets reaching 1.5–2 times the reflector depth. Tsvankin and Thomsen (1994) modified equation (1) by adding a denominator to the quartic moveout term to make $t(X)$ convergent at $X \to \infty$:

$$t^2 = A_0 + A_2 X^2 + \frac{A_4 X^4}{1 + A X^2}, \tag{3}$$

where the additional coefficient A depends on the horizontal group velocity V_{hor}:

$$A = \frac{A_4}{V_{hor}^{-2} - V_{nmo}^{-2}}. \tag{4}$$

Equation (3) was originally derived for VTI media, but its generic form makes it suitable for anisotropic media of any symmetry. For example, Al-Dajani and Tsvankin (1998) obtained the exact parameter A_4 for a horizontal HTI layer and used it to extend equation (3) to azimuthally dependent P-wave moveout in HTI media. Although equation (3) may be inadequate for pure (nonconverted) S-waves (Tsvankin and Thomsen, 1994), it provides a simple and numerically efficient way for modeling the reflection traveltimes of P-waves and, for relatively simple models, converted PS-waves (Tsvankin, 2001).

For horizontally layered anisotropic media above the reflector, the azimuthally varying NMO velocity (or A_2) can be determined from the generalized Dix formula of Grechka et al. (1999) that operates with interval NMO ellipses. Grechka and Tsvankin (2002) extended this Dix-type averaging equation to anisotropic media with arbitrarily heterogeneous overburden. The velocity V_{hor} used in equation (4) to define the coefficient A is found by averaging the interval horizontal velocities (Tsvankin, 2001).

Therefore, the key issue in applying equation (3) to modeling and inversion of reflection moveout is to derive the corresponding quartic moveout coefficient A_4. The dependence of A_4 on the medium parameters also yields valuable analytic insight into the properties of nonhyperbolic moveout.

GENERAL EXPRESSION FOR THE QUARTIC MOVEOUT COEFFICIENT A_4

Here, we present an exact expression for the quartic moveout coefficient in arbitrarily anisotropic, heterogeneous media (Figure 1). The derivation (see Appendix A) is based on expanding the two-way traveltime in a Taylor series in half-offset and applying the so-called normal-incidence-point (NIP) theorem (Chernjak and Gritsenko, 1979; Hubral and Krey, 1980; Fomel and Grechka, 2001) that helps to relate the Taylor series coefficients to the spatial derivatives of the traveltime between the common midpoint and the reflector. The quartic moveout coefficient A_4 in its most general form can be written as (Appendix A)

$$A_4(\mathbf{L}) = \frac{1}{16}\left[\frac{\partial^2\tau}{\partial y_k \partial y_l}\frac{\partial^2\tau}{\partial y_m \partial y_n} + \frac{\tau_0}{3}\frac{\partial^4\tau}{\partial y_k \partial y_l \partial y_m \partial y_n} - \tau_0 \frac{\partial^3\tau}{\partial y_k \partial y_l \partial x_i}\left(\frac{\partial^2\tau}{\partial x_i \partial x_j}\right)^{-1}\frac{\partial^3\tau}{\partial x_j \partial y_m \partial y_n}\right] L_k L_l L_m L_n, \quad (5)$$

where $\mathbf{L} = [\cos\alpha, \sin\alpha, 0]$ is a unit vector parallel to the CMP line SR (Figure 1), $\mathbf{y}$ defines the CMP location, τ is the one-way traveltime between the CMP and point $\mathbf{x}$ on the reflector, and τ_0 is the one-way zero-offset time. The traveltime derivatives are evaluated at the zero-offset reflection point $\mathbf{x}^{(0)}$ corresponding to the CMP location $\mathbf{y}$. Summation over repeated indices from one to two is implied.

Equation (5) was obtained without making specific assumptions about the anisotropy or heterogeneity of the model; also, it is generally valid for reflectors of irregular shape. However, our derivation assumes that the traveltime can be differentiated with respect to the spatial coordinates near the common midpoint, which is not the case, for example, in shadow zones. The Taylor series expansion for reflection traveltime may also break down for models with strong lateral velocity variations (Grechka and Tsvankin, 1998b) and in the vicinity of caustics. Nonetheless, for sufficiently smooth subsurface models commonly used in seismology, equation (5) is expected to give an accurate representation of the quartic moveout coefficient and the magnitude of nonhyperbolic moveout.

The form of the azimuthal dependence of the coefficient A_4 in equation (5) is governed by the derivatives of the traveltime τ with respect to the coordinates of the common midpoint $\mathbf{y}$ and point $\mathbf{x}$ on the reflector. For relatively simple models, τ can be expressed explicitly as a function of $\mathbf{y}$ and $\mathbf{x}$, and the derivatives in equation (5) can be evaluated in closed form (e.g., see Appendix B). However, if the medium is laterally heterogeneous and/or has a low anisotropic symmetry, it is convenient to express equation (5) in terms of the horizontal slowness component of the zero-offset ray (Cohen, 1998; Grechka et al., 1999). Most importantly, all derivatives in equation (5) can be evaluated using quantities computed during the tracing of the zero-offset ray.

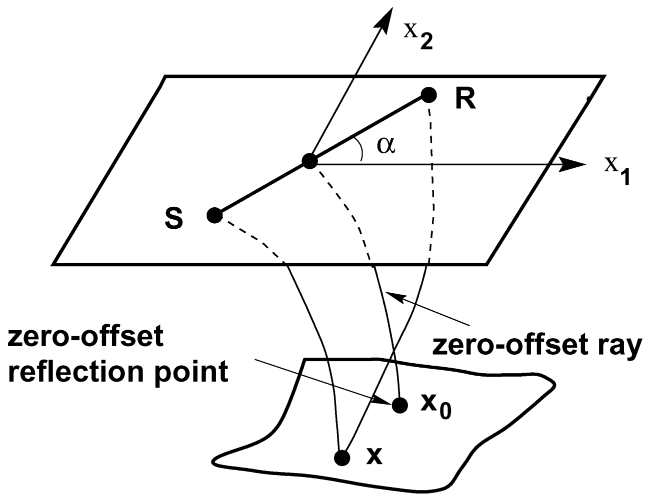

FIG. 1. Reflection traveltimes from an irregular interface beneath an arbitrarily anisotropic, heterogeneous medium are recorded in a multiazimuth CMP gather. The quartic moveout coefficient A_4 varies with the azimuth α of the CMP line. The derivation of A_4 in Appendix A accounts for reflection-point dispersal.

QUARTIC COEFFICIENT IN A HOMOGENEOUS TTI LAYER

Although equation (5) is completely general, the analysis hereafter is restricted to a homogeneous TTI layer overlaying a plane dipping reflector (Figure 2). Furthermore, we assume that the symmetry axis is confined to the dip plane of the reflector, which is typical for dipping TI formations (e.g., shales) in fold-and-thrust belts (Isaac and Lawton, 1999) or near salt domes (Tsvankin, 1997).

Hyperbolic reflection moveout and the dependence of NMO velocity on the anisotropic parameters for the tilted TI model was discussed by Tsvankin (1997, 2001) and Grechka and Tsvankin (2000). Following Tsvankin (1997), we parameterize the medium by the symmetry-direction velocities of P-waves (V_{P0}) and S-waves (V_{S0}), and Thomsen's anisotropic coefficients ϵ, δ, and γ specified with respect to the symmetry axis. In other words, the parameters are defined by the VTI expressions in the rotated coordinate system whose x_3-axis is aligned with the axis of symmetry. The tilt ν of the symmetry axis is considered positive if the axis points towards the reflector (i.e., if the symmetry axis and the reflector normal deviate from the vertical in the same direction).

Since the dip plane of the reflector contains the symmetry axis of the overburden, it represents a vertical symmetry

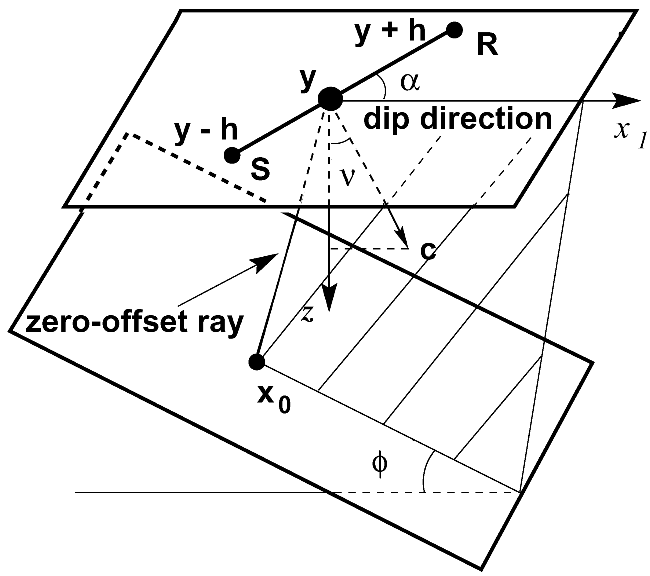

FIG. 2. Reflected wave is recorded above a homogeneous TI layer with a plane dipping lower boundary. The symmetry axis (vector $\mathbf{c}$) is contained in the dip plane $[x_1, z]$ but may be tilted away from the vertical at an arbitrary angle ν. The reflector dip is denoted by ϕ.

plane for the whole model. Therefore, the dip and strike directions of the reflector determine "the principal axes" of the azimuthally-varying quartic moveout coefficient A_4. Below, we use equation (5) to study the functional form of A_4 in a TTI layer and its dependence on reflector dip and anisotropic parameters. While the exact equation for the quartic coefficient is suitable for computational purposes, it does not provide analytic insight into the dependence of A_4 on the model parameters. As demonstrated in Appendix B, significant simplification can be achieved by applying the weak-anisotropy approximation and linearizing equation (5) in the anisotropic parameters.

Although the discussion of the weak-anisotropy results below is formally limited to P-waves, any kinematic signature of SV-waves (i.e., of the mode polarized in the plane formed by the slowness vector and the symmetry axis) for weak transverse isotropy can be obtained from the corresponding P-wave signature by making the following substitutions: $V_{P0} \to V_{S0}$, $\delta \to \sigma$, and $\epsilon \to 0$ (Tsvankin, 2001). The parameter $\sigma \equiv (V_{P0}/V_{S0})^2(\epsilon - \delta)$ is fully responsible for SV-wave velocity variations in weakly anisotropic TI media.

The linearized P-wave quartic moveout coefficient in a TTI layer is given by (Appendix B)

$$A_4^{\mathrm{TTI}} = -\frac{2\eta}{t_{P0}^2 V_{P0}^4} F(\alpha, \phi, \nu), \tag{6}$$

where the function F is defined in equation (B-14), t_{P0} is the two-way zero-offset P-wave traveltime, α is the azimuth of the CMP line measured from the dip plane, and ϕ is the reflector dip.

Figure 3 shows that the linearized equation (6) is sufficiently close to the exact quartic coefficient for relatively small values of the anisotropic parameters. The diamonds in Figure 3 correspond to the coefficient A_4 obtained by least-squares fitting of a quartic polynomial to reflection traveltimes generated by anisotropic ray tracing. Evidently, equation (6) (solid curve) provides a good approximation to the best-fit values of A_4 for the full range of azimuths. As demonstrated by Tsvankin and Thomsen (1994), the weak-anisotropy approximation may rapidly lose its accuracy with increasing parameters ϵ and δ. However, equation (6) can still be used to study the azimuthal signature of nonhyperbolic moveout in tilted TI media.

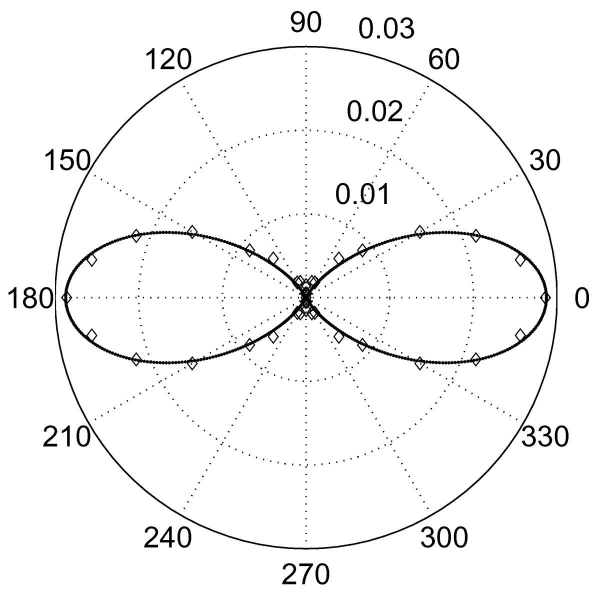

FIG. 3. Accuracy of the linearized equation for the coefficient A_4 in a tilted TI layer. The diamonds mark the magnitude of A_4 obtained for each azimuth (numbers on the perimeter) by fitting a quartic polynomial to the ray-traced $t^2(x^2)$–curve on the spreadlength $X_{\max} = 1.2z$, where $z = 1$ km is the reflector depth. The solid line is the weak-anisotropy approximation (6). The model parameters are $V_{P0} = 1$ km/s, $\epsilon = 0.1$, $\delta = 0.025$, $\nu = 80^\circ$, and $\phi = 0^\circ$.

ANALYSIS OF THE APPROXIMATE COEFFICIENT A_4 IN A TTI LAYER

It is clear from equation (6) that regardless of the tilt of the symmetry axis and reflector dip, nonhyperbolic moveout of P-waves for weak transverse isotropy is controlled by the anellipticity coefficient η, rather than by ϵ and δ individually. If the medium is elliptical ($\eta = 0$), A_4 vanishes, and reflection moveout becomes purely hyperbolic. This is a general result valid for an elliptically anisotropic layer with any strength of the anisotropy (Uren et al., 1990).

Dip component

Equations (6) and (B-14) can be used to find the coefficients A_4 in the dip and strike directions. On the dip line ($\alpha = 0^\circ$),

$$A_{4,\mathrm{dip}}^{\mathrm{TTI}}(\phi) = -\frac{2\eta}{t_{P0}^2 V_{P0}^4} \cos^3\phi \cos(4\nu - 3\phi). \tag{7}$$

Note that the quartic coefficient is proportional to $\cos^3\phi$, and the magnitude of nonhyperbolic moveout has a rapidly decreasing trend with dip [the influence of the term $\cos(4\nu - 3\phi)$ is discussed below]. Equation (7), however, becomes less accurate for near-vertical reflectors because when ϕ is close to 90°, several terms involving anisotropic coefficients can no longer be treated as small. Evaluation of the exact equation (5) shows that unless the symmetry axis is vertical or horizontal, A_4 for a vertical reflector ($\phi = 90^\circ$) is relatively small but usually does not go to zero. Also, note that for large dips and certain relative positions of the symmetry axis and the reflector normal (typically, if the symmetry axis is tilted toward the reflector), reflection events may not exist at the surface at all (Tsvankin, 1997, 2001).

According to equation (7), the dip-line quartic moveout coefficient (and, therefore, nonhyperbolic moveout as a whole) vanishes if $\cos(4\nu - 3\phi) = 0$, or $(4\nu - 3\phi) = n\pi/2$ ($n = \pm 1, \pm 3, \pm 5, \ldots$). In the special case of VTI media ($\nu = 0$), the dip line A_4 goes to zero for a dip of 30° (see a more detailed discussion of the VTI model below).

For a fixed reflector dip, $\cos(4\nu - 3\phi)$ goes to zero for two different values of the tilt ν between 0° and 90°, which is in good agreement with the computations of analytic (NMO) and finite-spread moveout velocity in Tsvankin (1995, 2001). Hence, the absence or low magnitude of dip-line nonhyperbolic moveout in nonelliptical ($\eta \neq 0$) TTI media may be used to constrain the relationship between the reflector dip and the tilt of the symmetry axis.

Equation (7) is written in terms of reflector dip that cannot be estimated from surface reflection data unless the velocity model is known. Therefore, for purposes of anisotropic parameter estimation, it is more convenient to rewrite the quartic coefficient as a function of the horizontal component p of the slowness vector associated with the zero-offset ray (e.g., Alkhalifah and Tsvankin, 1995). The horizontal slowness component, or the ray parameter, determines the slope of reflections on zero-offset (or stacked) sections and can be measured directly from surface data.

Substituting the ray parameter $p = \sin\phi / V(\phi)$ [$V(\phi)$ is the phase velocity at the dip angle] into equation (7) yields

$$A_{4,\mathrm{dip}}^{\mathrm{TTI}}(p) = \frac{8\eta(1-y)^3}{t_{P0}^2\left[V_{\mathrm{nmo}}^{\mathrm{TTI}}(0)\right]^4}\left[\left(y - \frac{1}{4}\right)\sqrt{1-y}\cos 4\nu + \left(y - \frac{3}{4}\right)\sqrt{y}\sin 4\nu\right], \quad (8)$$

where

$$y \equiv p^2\left[V_{\mathrm{nmo}}^{\mathrm{TTI}}(0)\right]^2, \quad (9)$$

and $V_{\mathrm{nmo}}^{\mathrm{TTI}}(0)$ is the NMO velocity from a horizontal reflector. Hence, $A_{4,\mathrm{dip}}^{\mathrm{TTI}}$ expressed as a function of p depends on three parameters: $V_{\mathrm{nmo}}^{\mathrm{TTI}}(0)$, η, and ν [or $V_{\mathrm{nmo}}^{\mathrm{TTI}}(0)$, $(\eta\cos 4\nu)$, and $(\eta\sin 4\nu)$].

In anisotropic parameter estimation, nonhyperbolic moveout is used in combination with the NMO velocity (e.g., Alkhalifah, 1997; Grechka and Tsvankin, 1998a). The dip-line P-wave NMO velocity for weakly anisotropic TTI media was given by Tsvankin (1997, 2001) as a function of the dip ϕ. Rewriting his result through the ray parameter p yields

$$V_{\mathrm{nmo}}^{\mathrm{TTI}}(p) = \frac{V_{\mathrm{nmo}}^{\mathrm{TTI}}(0)}{\sqrt{1-y}}[1 + \eta f\cos 4\nu - \eta g\sin 4\nu], \quad (10)$$

where

$$f \equiv \frac{y}{1-y}(6 - 9y + 4y^2) \quad (11)$$

and

$$g \equiv \sqrt{\frac{y}{1-y}}(3 - 7y + 4y^2). \quad (12)$$

For vertical transverse isotropy ($\nu = 0$), equation (10) reduces to the expression derived by Alkhalifah and Tsvankin (1995).

Evidently, both the dip-line NMO velocity and the quartic moveout coefficient are fully governed by the same parameter combinations: $V_{\mathrm{nmo}}^{\mathrm{TTI}}(0)$, $(\eta\cos 4\nu)$, and $(\eta\sin 4\nu)$. In principle, it may be possible to estimate those three combinations if V_{nmo} and A_4 are measured on the dip line for two different dips. The high level of structural complexity in overthrust areas or near salt domes may be sufficient for performing this type of inversion. However, as discussed by Grechka and Tsvankin (1998a), the trade-off between V_{nmo} and A_4 typically leads to a substantial uncertainty in the quartic coefficient. Also, according to Tsvankin (1997, 2001), the weak-anisotropy approximation for NMO velocity loses accuracy for the anisotropic coefficients reaching 0.15–0.2, and the exact V_{nmo} becomes dependent on the individual values of ϵ and δ.

Strike component

The strike component of the quartic moveout coefficient can be obtained by substituting $\alpha = 90^\circ$ into equation (6):

$$A_{4,\mathrm{strike}}^{\mathrm{TTI}}(\phi) = -\frac{2\eta}{t_{P0}^2 V_{P0}^4}\cos^4(\phi - \nu). \quad (13)$$

Both the dip and strike components of A_4 are proportional to η, but their dependencies on the reflector dip ϕ and the symmetry-axis tilt ν are different. Equation (13) shows that $A_{4,\mathrm{strike}}^{\mathrm{TTI}}$ goes to zero only if the symmetry axis is perpendicular to the reflector normal (i.e., the symmetry axis is confined to the reflecting plane). For example, if the reflector is vertical ($\phi = 90^\circ$), the strike-line quartic coefficient vanishes for VTI media ($\nu = 0^\circ$). Indeed, for such a model, reflected rays are confined to the horizontal (isotropy) plane where velocity is independent of angle, which makes reflection moveout for any azimuth (not just in the strike direction) purely hyperbolic.

In general, the fact that the dip and strike components of A_4 vanish for different combinations of ν and ϕ is favorable for a potential inversion procedure. The dip and strike components for some special cases of TI media are discussed in detail below.

Azimuthal dependence

Unlike NMO velocity that typically has a simple elliptical azimuthal dependence (Grechka and Tsvankin, 1998b), the variation of the quartic moveout coefficient with azimuth has a much more complicated character. The nonlinear relationship between A_4 and the angles ν, ϕ, and α [equation (6)] may lead to multiple zeros of the function $A_4(\alpha)$ whose positions strongly depend on both tilt ν and dip ϕ.

Figure 4 displays a polar plot with a typical azimuthal signature of A_4 in TTI media. Note that for the model considered

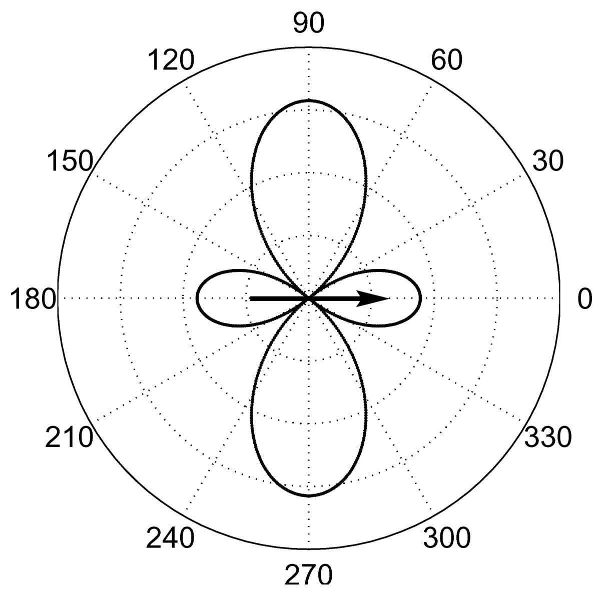

FIG. 4. Magnitude of the azimuthally-varying quartic moveout coefficient A_4 computed from equation (6). The tilt of the symmetry axis $\nu = 40^\circ$, the reflector dip $\phi = 15^\circ$; the other parameters change only the scale of the plot (intentionally undefined here). The azimuth is measured from the dip plane marked by the arrow.

here the quartic coefficient and the moveout signature as a whole are repeated in each quadrant since $A_4(\alpha)$ is symmetric with respect to both $\alpha = 0^\circ$ and $\alpha = 90^\circ$. Clearly, the quartic coefficient exhibits much more variability compared to the NMO ellipse, with zeros at azimuths of $\pm 38^\circ$. The sign of the coefficient A_4 changes from positive near the dip direction (for $\eta > 0$) to negative for the lobe centered at the strike direction.

These results indicate that the azimuthal signature of the quartic coefficient can provide useful information for anisotropic parameter estimation. In particular, the azimuthal directions of CMP lines with vanishing A_4 depend on certain combinations of ν and ϕ [equation (6)] and can be used to constrain the orientation of the symmetry axis. The variation of the sign of A_4 with azimuth is also sensitive to both ν and ϕ.

Special cases

Symmetry axis orthogonal to the reflector.—Because of the complicated structure of equation (6), here we focus on several special cases of practical importance. Models with the symmetry axis orthogonal to the reflector ($\phi = \nu$) are of particular importance for fold-and-thrust belts (e.g., the Canadian Foothills) where the anisotropy is caused by dipping TI shale layers. If $\phi = \nu$, the zero-offset ray is orthogonal to the reflector (as in isotropic media), and some isotropic relationships remain valid. For example, Tsvankin (1995, 2001) demonstrated that for $\phi = \nu$, the dip-line NMO velocity obeys the conventional (isotropic) cosine-of-dip dependence.

To study the azimuthal dependence of A_4, we substitute $\phi = \nu$ into equation (6):

$$A_4^{\mathrm{TTI}}(\phi = \nu) = -\frac{2\eta}{t_{P0}^2 V_{P0}^4}(1 - \sin^2\phi \cos^2\alpha)^2. \qquad (14)$$

According to equation (14), the quartic coefficient goes to zero when

$$|\cos\alpha| = \frac{1}{\sin\phi}. \qquad (15)$$

Condition (15) can be satisfied only on the dip line ($\alpha = 0^\circ$) of a vertical reflector ($\phi = 90^\circ$, which implies a horizontal symmetry axis). Away from the dip line, the coefficient A_4 for a vertical reflector varies as

$$A_4^{\mathrm{TTI}}(\phi = \nu = 90^\circ) = -\frac{2\eta}{t_{P0}^2 V_{P0}^4}\sin^4\alpha. \qquad (16)$$

If $\phi = \nu$, equations (7) and (13) [or equation (14)] yield the following expressions for the dip and strike components of A_4:

$$A_{4,\mathrm{dip}}^{\mathrm{TTI}}(\phi = \nu) = -\frac{2\eta}{t_{P0}^2 V_{P0}^4}\cos^4\phi, \qquad (17)$$

$$A_{4,\mathrm{strike}}^{\mathrm{TTI}}(\phi = \nu) = -\frac{2\eta}{t_{P0}^2 V_{P0}^4}. \qquad (18)$$

Equation (18), which shows that the strike-line component of A_4 is independent of dip (or tilt), is well known for weakly anisotropic VTI media and a horizontal reflector, when $\phi = \nu = 0$ (Tsvankin and Thomsen, 1994; Alkhalifah and Tsvankin, 1995). $A_{4,\mathrm{strike}}^{\mathrm{TTI}}$ has the same value for an HTI layer ($\nu = 90^\circ$) and a vertical reflector ($\phi = 90^\circ$). Since the strike line for this HTI model is perpendicular to the symmetry axis and reflected rays are horizontal, reflection moveout in the strike direction is identical to that for a VTI layer above a horizontal reflector.

Whereas the strike-line component of A_4 does not change with dip, the dip-line component is proportional to $\cos^4\phi$ [equation (17)]. Therefore, nonhyperbolic moveout for dipping reflectors rapidly decays away from the strike direction, even if the dip is relatively mild.

Dipping reflector beneath a VTI layer.—Setting the tilt ν of the symmetry axis in equation (6) to zero yields the weak-anisotropy approximation for the quartic coefficient in VTI media:

$$A_4^{\mathrm{VTI}} = -\frac{2\eta\cos^4\phi}{t_{P0}^2 V_{P0}^4}(1 - 4\sin^2\phi\cos^2\alpha). \qquad (19)$$

For a vertical reflector ($\phi = 90^\circ$), A_4 vanishes regardless of the azimuth of the CMP line because reflected rays are confined to the horizontal isotropy plane where velocity is constant and moveout is purely hyperbolic. If the reflector is horizontal ($\phi = 0^\circ$), the model as a whole is azimuthally isotropic, and the approximate A_4 is determined by equation (18). A discussion of the exact (i.e., not limited to weak anisotropy) quartic moveout coefficient of both P- and S-waves in horizontally layered VTI media can be found in Tsvankin (2001).

For a dipping reflector, the coefficient A_4 vanishes in azimuthal directions satisfying

$$|\cos\alpha| = \frac{1}{2\sin\phi}. \qquad (20)$$

If the dip is smaller than 30°, equation (20) does not have a solution, and A_4 has the same sign for the whole range of azimuthal directions (Figure 5). For a dip of 30°, A_4 goes to

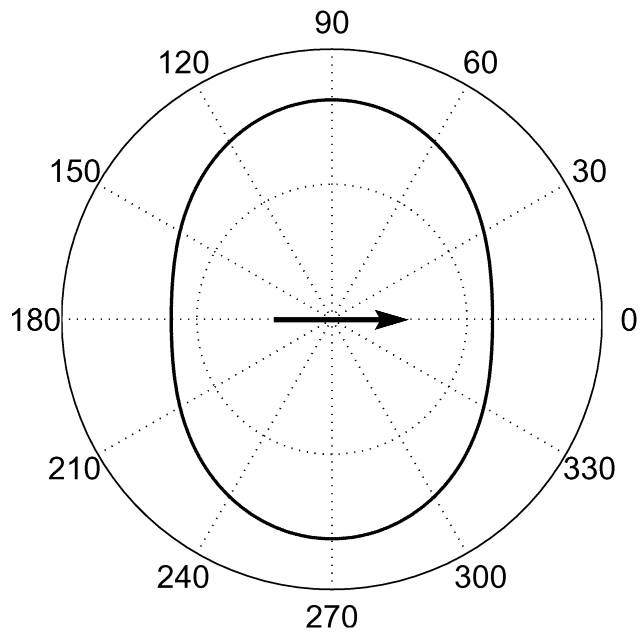

FIG. 5. Magnitude of the azimuthally-varying coefficient A_4 for a VTI layer computed from equation (19). Reflector dip is 15°; the dip direction is marked by the arrow. The parameter η is positive, and $A_4 < 0$ for all azimuths.

zero only for a single azimuth $\alpha = 0^\circ$ that corresponds to the dip plane (Figure 6). This analytic result is in good agreement with the numerical study of NMO velocity in Tsvankin (1995, 2001) who showed that the P-wave dip-line moveout approaches a hyperbola for reflector dips relatively close to 30°.

For $30^\circ < \phi < 90^\circ$, equation (20) yields two azimuths $\pm\alpha$ for which $A_4 = 0$. If the dip is equal to 45°, the quartic coefficient vanishes for $\alpha = \pm 45^\circ$ (Figure 7). The sign of A_4 for $\eta > 0$ is positive near the dip plane ($-45^\circ < \alpha < 45^\circ$) and negative near the strike direction.

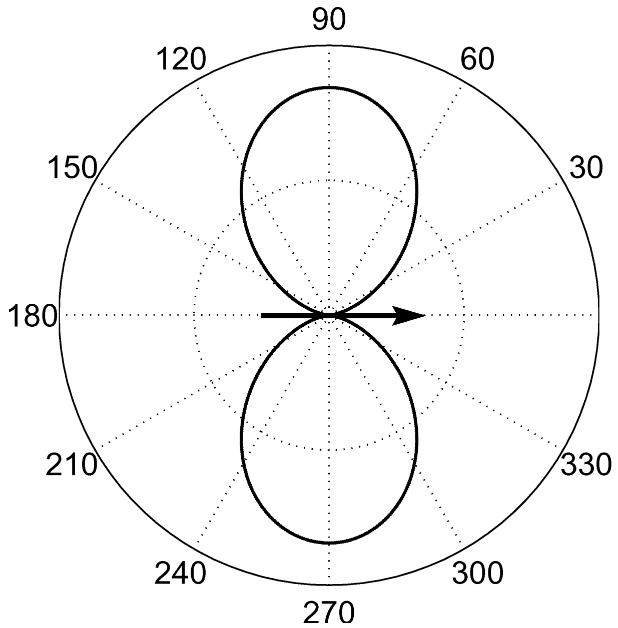

FIG. 6. Same as Figure 6, but the reflector dip is 30°.

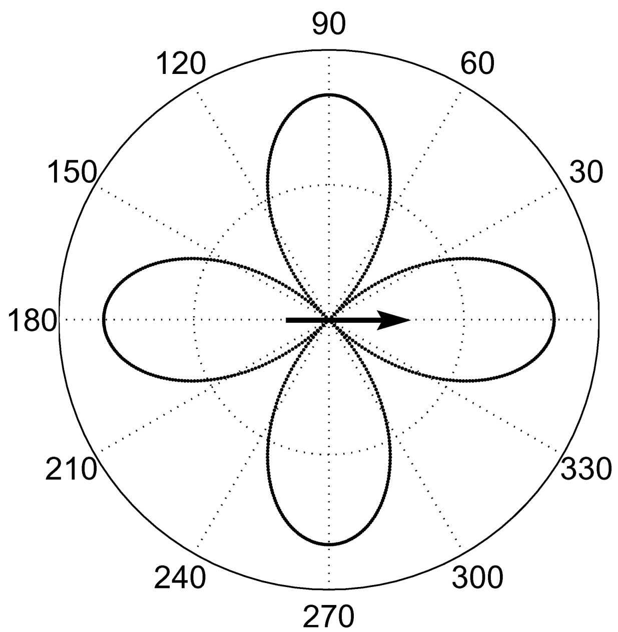

FIG. 7. Same as Figure 6, but the reflector dip is 45°.

Horizontal HTI layer.—For a horizontal HTI layer ($\nu = 90^\circ$ and $\phi = 0^\circ$), equation (6) reduces to

$$A_4^{\mathrm{HTI}} = -\frac{2\eta}{t_{P0}^2 V_{P0}^4} \cos^4 \alpha. \tag{21}$$

Equation (21) has the same azimuthal dependence ($\cos^4 \alpha$) as the exact expression for A_4 obtained by Al-Dajani and Tsvankin (1998). In the expression of Al-Dajani and Tsvankin (1998), however, the term multiplied with $\cos^4 \alpha$ corresponds to the exact quartic coefficient in the plane that contains the symmetry axis ($\alpha = 0^\circ$). The quartic coefficient vanishes in the isotropy plane orthogonal to the symmetry axis ($\alpha = 90^\circ$), where reflection moveout is purely hyperbolic.

DISCUSSION AND CONCLUSIONS

We have presented an exact expression for the quartic moveout coefficient A_4 valid for arbitrarily anisotropic, heterogeneous media. Unlike most existing methods, our approach does not require the model to have a horizontal symmetry plane and accounts for reflection-point dispersal on dipping or irregular interfaces. Substitution of the quartic coefficient into the general moveout equation of Tsvankin and Thomsen (1994) yields a good approximation for nonhyperbolic moveout of P-waves and, in some cases, mode-converted PS-waves in anisotropic media with moderate structural complexity.

It should be emphasized that all quantities needed to calculate the azimuthally-varying quartic coefficient can be obtained by tracing a single (zero-offset) ray. Computing the zero-offset ray is also sufficient to construct the NMO ellipse (i.e., the azimuthally-varying NMO velocity) responsible for short-spread moveout (Grechka et al., 1999; Grechka and Tsvankin, 2002). Therefore, our results can be used to model azimuthally dependent long-spread moveout in a computationally efficient way, without multioffset, multiazimuth ray tracing.

The general equation for A_4 was applied to study the properties of P-wave nonhyperbolic moveout in TI media with a tilted symmetry axis. The analysis was restricted to a homogeneous TI layer above a plane horizontal or dipping reflector; it was assumed that the symmetry axis is confined to the dip plane. To gain insight into the dependence of the quartic moveout coefficient on the model parameters, we simplified the exact expression by linearizing it in the anisotropic parameters. The derived weak-anisotropy approximation is proportional to the "anellipticity" parameter $\eta \approx \epsilon - \delta$, so the magnitude of nonhyperbolic moveout increases as the model deviates from elliptical ($\epsilon = \delta$).

Although the azimuthally varying coefficient $A_4(\alpha)$ is a rather complicated function of the reflector dip ϕ and the tilt ν of the symmetry axis, the expressions for A_4 in the "principal" (dip and strike) directions are relatively simple. In particular, the strike component of A_4 depends solely on the *difference* between the dip and tilt rather than on their individual values. The magnitude of the dip component is proportional to $\cos^3 \phi$, so it rapidly decreases with ϕ. For a fixed dip, the dip-line quartic coefficient vanishes for two values of the tilt between 0° and 90°.

The azimuthal signature of the quartic coefficient is quite different from the elliptical variation of NMO velocity. Although the function $A_4(\alpha)$ in tilted TI media is repeated in each quadrant, the quartic coefficient may vanish in one or more azimuthal directions. For weak anisotropy, the azimuthal positions of the zeros of the quartic coefficients and the signs of A_4 in different azimuthal sectors are governed by the tilt ν and reflector dip ϕ (η plays the role of a scaling coefficient). In realistic heterogeneous media, nonhyperbolic moveout is also caused by vertical and lateral velocity gradients, but anisotropy usually makes the most prominent contribution to A_4 (Alkhalifah, 1997). Therefore, the character of the azimuthal dependence of nonhyperbolic moveout over a medium containing a tilted TI layer should be well described by the equations given in this paper.

In the important special case of the symmetry axis orthogonal to the reflector ($\phi = \nu$), the strike-line A_4 is independent of dip (and tilt) and has the same value as in VTI media, while the dip-line A_4 decreases with dip as $\cos^4 \phi$. Therefore, the magnitude of nonhyperbolic moveout for $\phi = \nu$ is significant mostly for azimuthal directions close to the reflector strike.

For weakly anisotropic VTI media with typical positive η and mild reflector dips ($\phi < 30^\circ$), A_4 is negative for all azimuths, and its magnitude increases away from the dip direction. If the dip is equal to 30°, nonhyperbolic moveout in VTI media vanishes on the dip line, which agrees with existing numerical results (Tsvankin, 1995, 2001). If the dip exceeds 30°, A_4 goes to zero in two different azimuths that do not coincide with either dip or strike directions.

For purposes of anisotropic parameter estimation, moveout equations have to be rewritten in terms of the ray parameter p that can be determined from reflection slopes on zero-offset (or stacked) sections. The dip components of both A_4 and NMO velocity expressed through p depend on the same three parameter combinations involving η, ν, and the NMO velocity from a horizontal reflector. This result and the high sensitivity of the azimuthal signature of A_4 to the symmetry-axis orientation indicate that P-wave nonhyperbolic moveout may provide valuable information for velocity analysis in TTI media. Although the trade-off between $V_{\rm nmo}$ and A_4 makes quantitative estimates of the quartic coefficient relatively unstable (Grechka and Tsvankin, 1998a), the azimuthal variation of the sign of A_4 and the directions of vanishing or small nonhyperbolic moveout should be detectable from wide-azimuth reflection data.

ACKNOWLEDGMENTS

We are grateful to members of the A(nisotropy)-Team of the Center for Wave Phenomena (CWP), Colorado School of Mines, for helpful discussions. We thank Kurt Marfurt, associate editor of *Geophysics*, for his review of the manuscript. The support for this work was provided by the Consortium Project on Seismic Inverse Methods for Complex Structures at CWP and by the Chemical Sciences, Geosciences and Biosciences Division, Office of Basic Energy Sciences, U.S. Department of Energy.

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APPENDIX A

DERIVATION OF THE QUARTIC MOVEOUT COEFFICIENT

Here, we develop a general analytic expression for the quartic moveout coefficient A_4 by extending the approach employed by Grechka and Tsvankin (2002) in their analysis of NMO-velocity surfaces. Suppose the traveltime t of a certain pure (nonconverted) reflected wave is recorded on a CMP line parallel to the unit vector $\mathbf{L}$. The coordinates of the source $\mathbf{S}$ and receiver $\mathbf{R}$ are defined by the vectors $\mathbf{y}-\mathbf{h}$ and $\mathbf{y}+\mathbf{h}$ (Figure A-1), where $\mathbf{y}$ corresponds to the midpoint, and the half-offset vector $\mathbf{h}$ can be represented as

$$\mathbf{h} = h\mathbf{L} = h[L_1, L_2, 0]. \tag{A-1}$$

To obtain the quartic moveout coefficient, we expand the two-way traveltime in a Taylor series with respect to the half-offset h in the vicinity of the CMP location ($h=0$):

$$t(h,\mathbf{L}) = t_0 + \left.\frac{dt}{dh}\right|_{h=0} h + \left.\frac{d^2t}{dh^2}\right|_{h=0}\frac{h^2}{2!} + \left.\frac{d^3t}{dh^3}\right|_{h=0}\frac{h^3}{3!} + \left.\frac{d^4t}{dh^4}\right|_{h=0}\frac{h^4}{4!} + \cdots, \tag{A-2}$$

where t_0 is the zero-offset traveltime.

For a pure (nonconverted) reflection mode, t is an even function of h because the traveltime remains the same when the source and receiver are interchanged. Therefore, the odd derivatives of t can be dropped from equation (A-2), which leads to

$$t(h,\mathbf{L}) = t_0 + \left.\frac{d^2t}{dh^2}\right|_{h=0}\frac{h^2}{2!} + \left.\frac{d^4t}{dh^4}\right|_{h=0}\frac{h^4}{4!} + \cdots \tag{A-3}$$

To find the derivatives in equation (A-3), it is convenient to treat reflection traveltime for a fixed CMP location $\mathbf{y}$ as a function of $\mathbf{h}$ and the coordinates $[x_1, x_2, z(x_1, x_2)]$ of the reflection point $\mathbf{x}$. Since the specular reflection point is determined by the source and receiver positions, $t = t(\mathbf{h}, \mathbf{x}(\mathbf{h}))$.

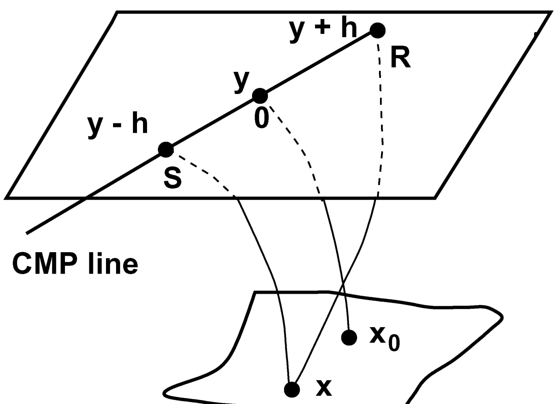

FIG. A-1. CMP line over an arbitrarily anisotropic, heterogeneous medium. The derivation of the quartic coefficient is based on expanding the two-way traveltime in the half-offset h.

Using this representation of traveltime and taking into account equation (A-1) yields

$$\frac{dt}{dh} = \frac{\partial t}{\partial h_k}L_k + \frac{\partial t}{\partial x_k}\frac{dx_k}{dh}, \quad (k = 1, 2). \tag{A-4}$$

Here and below, summation over repeated indices from one to two is implied. According to Fermat's principle, for the specular ray

$$\frac{\partial t}{\partial x_k} = 0, \tag{A-5}$$

and equation (A-4) takes the form

$$\frac{dt}{dh} = \frac{\partial t}{\partial h_k}L_k. \tag{A-6}$$

Evaluating the derivative of equation (A-6), we obtain

$$\frac{d^2t}{dh^2} = \frac{\partial^2 t}{\partial h_k \partial h_m}L_k L_m + \frac{\partial^2 t}{\partial h_k \partial x_m}\frac{dx_m}{dh}L_k, \tag{A-7}$$

where $k=1,2$ and $m=1,2$.

Differentiating equation (A-7) twice gives the following expression for the fourth derivative of t:

$$\begin{aligned}\frac{d^4t}{dh^4} = {} & \frac{\partial^4 t}{\partial h_p \partial h_k \partial h_m \partial h_n}L_p L_k L_m L_n \\ & + 3\frac{\partial^3 t}{\partial x_n \partial h_k \partial h_m}\frac{\partial^2 x_n}{\partial h^2}L_k L_m \\ & + \frac{\partial^4 t}{\partial x_p \partial h_k \partial h_m \partial h_n}\frac{\partial x_p}{\partial h}L_k L_m L_n \\ & + 2\frac{\partial^4 t}{\partial h \partial x_n \partial h_k \partial h_m}\frac{\partial x_n}{\partial h}L_k L_m \\ & + \frac{\partial^4 t}{\partial h \partial x_n \partial x_m \partial h_k}\frac{\partial x_n}{\partial h}\frac{\partial x_m}{\partial h}L_k \\ & + \frac{\partial^3 t}{\partial x_n \partial x_m \partial h_k}\frac{\partial}{\partial h}\left(\frac{\partial x_n}{\partial h}\frac{\partial x_m}{\partial h}\right)L_k \\ & + \frac{\partial^2 t}{\partial x_m \partial h_k}\frac{\partial^3 x_m}{\partial h^3}L_k \\ & + \frac{\partial^3 t}{\partial x_n \partial x_m \partial h_k}\frac{\partial x_n}{\partial h}\frac{\partial^2 x_m}{\partial h^2}L_k. \end{aligned} \tag{A-8}$$

Since not only the traveltime, but also the ray trajectory stays the same when the source and receiver are interchanged, the vector $\mathbf{x}$ is an even function of h:

$$\mathbf{x}(\mathbf{y}, h\mathbf{L}) = \mathbf{x}(\mathbf{y}, -h\mathbf{L}), \tag{A-9}$$

and

$$\left.\frac{d\mathbf{x}}{dh}\right|_{h=0} = \left.\frac{d^3\mathbf{x}}{dh^3}\right|_{h=0} = 0. \tag{A-10}$$

Taking equation (A-10) into account, the derivative d^4t/dh^4 [equation (A-8)] at the CMP location ($h = 0$) becomes

$$\left.\frac{d^4t}{dh^4}\right|_{h=0} = \left.\frac{\partial^4 t}{\partial h_p \partial h_k \partial h_m \partial h_n}\right|_{h=0} L_p L_k L_m L_n + 3 \left.\frac{\partial^3 t}{\partial x_n \partial h_k \partial h_m}\right|_{h=0} \left.\frac{\partial^2 x_n}{\partial h^2}\right|_{h=0} L_k L_m. \quad \text{(A-11)}$$

Introducing the offset X ($X = 2h$) and the one-way traveltime τ from the surface to the reflector and using the results of Fomel and Grechka (2001), equation (A-11) can be rewritten as

$$\left.\frac{d^4t}{dX^4}\right|_{X=0} = \frac{1}{8}\left[\frac{\partial^4 \tau}{\partial y_p \partial y_k \partial y_m \partial y_n} L_p L_k L_m L_n + 3 \frac{\partial^3 \tau}{\partial x_n \partial y_k \partial y_m}\left(\left.\frac{\partial^2 x_n}{\partial h^2}\right|_{h=0}\right) L_k L_m\right], \quad \text{(A-12)}$$

where the derivatives with respect to x_i and y_i are evaluated at the zero-offset reflection point and the CMP location (i.e., for the zero-offset ray).

To represent the derivatives $(\partial^2 x_n/\partial h^2)$ in terms of the traveltime, we use Fermat's principle expressed in the following form (Grechka and Tsvankin, 2002):

$$\frac{\partial t}{\partial x_n} = 0, \quad (n = 1, 2). \quad \text{(A-13)}$$

Differentiating equation (A-13) twice with respect to h yields

$$\frac{\partial^3 t}{\partial h_p \partial h_k \partial x_n} L_p L_k + \frac{\partial^2 t}{\partial x_k \partial x_n}\left(\left.\frac{\partial^2 x_k}{\partial h^2}\right|_{h=0}\right) = 0 \quad \text{(A-14)}$$

and

$$\frac{\partial^3 \tau}{\partial y_p \partial y_k \partial x_n} L_p L_k + \frac{\partial^2 \tau}{\partial x_k \partial x_n}\left(\left.\frac{\partial^2 x_k}{\partial h^2}\right|_{h=0}\right) = 0. \quad \text{(A-15)}$$

As before, all derivatives in equations (A-14) and (A-15) are computed for the zero-offset ray. Two equations (A-15) corresponding to $n = 1, 2$ can be solved for the derivatives $(\partial^2 x_1/\partial h^2)$ and $(\partial^2 x_2/\partial h^2)$ at $h = 0$. Substituting the result into equation (A-12), we find

$$\left.\frac{d^4t}{dX^4}\right|_{X=0} = \frac{1}{8}\left[\frac{\partial^4 \tau}{\partial y_p \partial y_k \partial y_m \partial y_n} L_p L_k L_m L_n - 3 \frac{\partial^3 \tau}{\partial x_i \partial y_k \partial y_m}\left(\frac{\partial^2 \tau}{\partial x_i \partial x_j}\right)^{-1} \frac{\partial^3 \tau}{\partial x_j \partial y_p \partial y_n} L_k L_m L_p L_n\right]. \quad \text{(A-16)}$$

After the fourth traveltime derivative has been obtained, the quartic moveout coefficient can be determined from the Taylor series (A-3). Introducing the offset $X = 2h$ into equation (A-3) and squaring the first three terms of the series leads to

$$t^2(X, \mathbf{L}) \approx \left(t_0 + \left.\frac{d^2t}{dX^2}\right|_{X=0} \frac{X^2}{2!} + \left.\frac{d^4t}{dX^4}\right|_{X=0} \frac{X^4}{4!}\right)^2. \quad \text{(A-17)}$$

Keeping only the quartic and lower-order terms in X transforms equation (A-17) into

$$t^2(X, \mathbf{L}) \approx A_0 + A_2 X^2 + A_4 X^4, \quad \text{(A-18)}$$

where $A_0 = t_0^2 = 4\tau_0^2$ denotes the squared two-way zero-offset traveltime (τ_0 is the one-way zero-offset time),

$$A_2 = t_0 \left.\frac{d^2t}{dX^2}\right|_{X=0} = \tau_0 \frac{\partial^2 \tau}{\partial y_k \partial y_m} L_k L_m = \frac{1}{V^2_{\text{nmo}}(\mathbf{L})} \quad \text{(A-19)}$$

is the quantity reciprocal to the squared NMO velocity on the CMP line $\mathbf{L}$ (Grechka and Tsvankin, 1998b), and

$$A_4 = \frac{\tau_0}{6} \left.\frac{d^4t}{dX^4}\right|_{X=0} + \frac{1}{4}\left(\left.\frac{d^2t}{dX^2}\right|_{X=0}\right)^2. \quad \text{(A-20)}$$

Substituting the derivatives d^4t/dX^4 from equation (A-16) and d^2t/dX^2 from equation (A-19) into equation (A-20) yields the final expression for the quartic coefficient:

$$A_4 = \frac{1}{16}\left[\frac{\partial^2 \tau}{\partial y_k \partial y_l}\frac{\partial^2 \tau}{\partial y_m \partial y_n} + \frac{\tau_0}{3}\frac{\partial^4 \tau}{\partial y_k \partial y_l \partial y_m \partial y_n} - \tau_0 \frac{\partial^3 \tau}{\partial y_k \partial y_l \partial x_i}\left(\frac{\partial^2 \tau}{\partial x_i \partial x_j}\right)^{-1} \frac{\partial^3 \tau}{\partial x_j \partial y_m \partial y_n}\right] L_k L_l L_m L_n. \quad \text{(A-21)}$$

APPENDIX B

WEAK-ANISOTROPY APPROXIMATION FOR THE P-WAVE QUARTIC MOVEOUT COEFFICIENT IN TTI MEDIA

We consider a homogeneous TI layer above a plane dipping reflector and assume that the symmetry axis (unit vector **c**) lies in the dip plane (Figure 2). Then the zero-offset ray should be confined to the dip plane that represents a vertical plane of symmetry for the whole model. Without losing generality, the x_1-axis can be aligned with the dip direction, so that the vector **c** is given by

$$\mathbf{c} = [\sin \nu, 0, \cos \nu]. \quad \text{(B-1)}$$

The one-way traveltime τ between the common-midpoint **y** and the reflector in a homogeneous medium is simply

$$\tau(y_1, y_2, x_1, x_2) = \frac{\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + z^2(x_1, x_2)}}{V_G(y_1, y_2, x_1, x_2)}. \quad \text{(B-2)}$$

Here $z(x_1, x_2)$ defines the reflecting plane, and $V_G(y_1, y_2, x_1, x_2)$ is the group velocity. Using the weak-anisotropy approximations for the P-wave group velocity and group angle in TI media (e.g., Tsvankin, 2001), V_G can be found as

$$V_G = \frac{V_{P0}}{4}\{4 + m[-\delta - \epsilon - \eta \sin^2 a \sin^2 b - \eta \cos 2\nu\,(2\cos^2 b \sin^2 a + \sin^2 a \sin^2 b - 1) + \eta \cos b \sin 2a \sin 2\nu]\}, \quad \text{(B-3)}$$

where

$$\sin a \equiv \frac{\sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2}}{\sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2 + z^2}}, \quad \text{(B-4)}$$

$$\cos a \equiv \frac{z}{\sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2 + z^2}}, \quad \text{(B-5)}$$

$$\sin b \equiv \frac{(y_2 - x_2)}{\sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2}}, \quad \text{(B-6)}$$

$$\cos b \equiv \frac{(y_1 - x_1)}{\sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2}}, \quad \text{(B-7)}$$

$$m \equiv -1 - \sin^2 a \sin^2 b - \cos 2\nu\,(-1 + 2\sin^2 a \cos^2 b + \sin^2 a \sin^2 b) + \cos b \sin 2a \sin 2\nu. \quad \text{(B-8)}$$

The parameters used in equation (B-3) are introduced in the main text. Since the zero-offset traveltime needs to satisfy Fermat's principle, the minimum value of τ corresponds to the coordinates $x_1^{(0)}$ and $x_2^{(0)}$ of the zero-offset reflection point. This implies that the derivatives of $\tau(y_1, y_2, x_1, x_2)$ with respect to x_1 and x_2 should vanish at the point $[x_1^{(0)}, x_2^{(0)}]$:

$$\left.\frac{\partial \tau(y_1, y_2, x_1, x_2)}{\partial x_1}\right|_{[x_1^{(0)}, x_2^{(0)}]} = 0, \quad \text{(B-9)}$$

$$\left.\frac{\partial \tau(y_1, y_2, x_1, x_2)}{\partial x_2}\right|_{[x_1^{(0)}, x_2^{(0)}]} = 0. \quad \text{(B-10)}$$

Equations (B-9) and (B-10) can be used to relate the CMP coordinates y_1 and y_2 to $x_1^{(0)}$ and $x_2^{(0)}$. Substituting equations (B-2) and (B-3) into equations (B-9) and (B-10) and dropping quadratic and higher-order terms in the anisotropic coefficients yields

$$y_1 = z\,[\epsilon - \eta \cos 2(\phi - \nu)]\,\frac{\sin 2(\phi - \nu)}{\cos^2 \phi} + z \tan \phi + x_1^{(0)}, \quad \text{(B-11)}$$

$$y_2 = x_2^{(0)}. \quad \text{(B-12)}$$

Equation (B-12) confirms the obvious fact that the zero-offset ray is confined to the dip plane $x_2 =$ constant; if the CMP lies on the x_1-axis (Figure 2), $x_2^{(0)} = y_2 = 0$.

Using equations (B-2), (B-11), and (B-12) to evaluate the derivatives in equation (A-21), we obtain the following linearized approximation for the P-wave quartic moveout coefficient:

$$A_4^{\mathrm{TTI}} = -\frac{2\eta}{t_{P0}^2 V_{P0}^4} F(\alpha, \phi, \nu), \quad \text{(B-13)}$$

where

$$\begin{aligned} F(\alpha, \phi, \nu) = \frac{1}{128}[&18 - 24\cos 2\alpha + 6\cos 4\alpha \\ &+ 8\cos(6\phi - 4\nu) + 4\cos 2(\alpha - 2\nu) \\ &- 4\cos(4\alpha - 2\nu) + 24\cos 2(\phi - 2\nu) \\ &+ 12\cos 2(\alpha + \phi - 2\nu) + 8\cos 2(\alpha + 2\phi - 2\nu) + 4\cos 2(\alpha + 3\phi - 2\nu) \\ &+ \cos 4(\alpha - \nu) + 32\cos 2(\phi - \nu) \\ &+ 32\cos 4(\phi - \nu) - 16\cos 2(\alpha + \phi - \nu) \\ &+ 8\cos 2\nu + 6\cos 4\nu \\ &+ \cos 4(\alpha + \nu) - 4\cos 2(2\alpha + \nu) \\ &- 16\cos 2(\alpha - \phi + \nu) + 4\cos 2(\alpha + 2\nu) \\ &+ 4\cos 2(\alpha - 3\phi + 2\nu) + 8\cos 2(\alpha - 2\phi + 2\nu) + 12\cos 2(\alpha - \phi + 2\nu)]. \end{aligned} \quad \text{(B-14)}$$