Integrating pressure-transient experiments and seismic reservoir characterization

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ABSTRACT
Pressure-transient experiments performed by engineers perturb the pore pressure of the reservoir and test the compliance of the rock's pores. Seismic experiments performed by geophysicist perturb the confining pressure and test the compliance of the bulk rock volume. Since the elasticity theory provides relations between the pore and bulk compressibilities, well test and seismic data analysis can be used jointly to characterize the elasticity of the reservoir rocks. Gassmann's equations are rewritten to take advantage of the information contained in the storage capacity measured from pressure-transient tests. In double porosity fractured reservoirs the ratio of the storage capacity of the fractures to the total storage capacity of the rock, can be used to estimate the fracture system's normal compliance and fracture density.

Key words: well test analysis, storage capacity, Gassman's equation, fractures

INTRODUCTION

Well test analysis is the generic name given to the study of pressure and flow-rate data measured at the well location. If Darcy's law is valid and the rock behaves elastically, the diffusion equation describes fluid flow through the reservoir, providing estimates of storage capacity, average permeability and average pressure, (Da Prat, 1990). The condition of elasticity inherent in the diffusion equation links well test measurements to the pore space compressibility of the rock. Since the compressibility of the pore space is a function of the mineral modulus and dry bulk modulus of the rock (Zimmerman, 1991), well test measurements can be related to some of the elastic parameters that are estimated from seismic data. Therefore, well test analysis can provide a quantitative link between geophysical and engineering measurements that is useful for reservoir characterization.

Both well test and seismic propagation analysis suffer from non-uniqueness of the solutions that model a given reservoir. By estimating some of the elastic parameters of the rock from both a flow experiment and seismic data, one can reduce the non-uniqueness of the solutions by correlating the outcome of the two independent analyses.

In this paper, Zimmerman's (1991) rock compressibility relations are used to show how the rock's storage capacity is a function of the mineral and dry rock compressibility. If the rock is isotropic, storage capacity estimates from pressure-transient tests can be used in Gassmann's (1951) equations to predict changes in the bulk modulus of the rock with saturation, *without knowing the fluid compressibility and the rock porosity.* If the rock is fractured and anisotropic, the application of Schoenberg's linear slip theory (Schoenberg and Douma, 1988; Schoenberg and Sayers, 1995) yields a relation between the normal compliance of the fracture system and the ratio of the fracture storage capacity to total rock storage capacity (storage capacity ratio). Since the normal compliance of the fracture system may be estimated from seismic data (Bakulin et al., 2000), and the storage capacity ratio can be measured from well test data, the derived relation allows a quantitative comparison between independent experiments.

The developed theory is applied to the analysis of pressure-transient data from the fractured Mississippian carbonate reservoir at Weyburn field, Saskatchewan, Canada. There is extensive proof from production, core

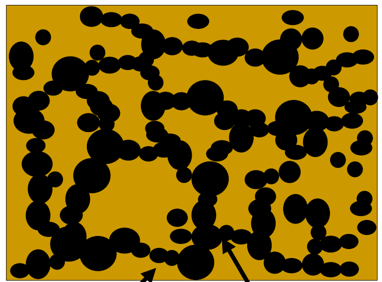

Figure 1. Schematic representation of an isotropic, single porosity rock. The ISP rock may be modeled assuming the pores are all spherical or of arbitrary shape with random orientations.

and borehole imager data that the fracture network is important in controlling flow in the reservoir. Application of the theory provides estimates of the normal fracture compliance and fracture density from the storage capacity ratio. Furthermore, normal fracture compliance values derived from seismic data can be used to predict the storage capacity ratio where no wells are available.

Even though there exists a variety of methods and experiments from which formation storage capacity can be estimated (Lee, 1982; Da Pratt, 1990, Raghavan, 1993), this paper will concentrate on single well, pressure-transient analysis in which the well is produced at a constant rate. However, the link between the well test parameters and the rock's elastic parameters will be valid for any other type of well test experiment.

PART I: ISOTROPIC SINGLE POROSITY ROCK

The first model to be considered is defined as the **isotropic, single porosity (ISP)** rock, which is composed of a mineral phase and a single porous phase. The mineral phase is assumed to be elastically isotropic and homogeneous, and the pores are assumed to be randomly oriented. This condition implies that the porous rock will be elastically **isotropic** and that it can be modeled as a collection of spherical pores or randomly oriented pores in an isotropic background material (see Figure 1).

The flow properties of the isotropic, single porosity rock, require that all the pores that account for the flow account for the storage capacity of the rock. This means that the storage capacity measured from a flow experiment corresponds to the connected pores of the rock. The storage capacity contains information about the pore and fluid compressibility, providing the link between the well test experiment and the elasticity of the rock.

Measuring the storage capacity

In the **ISP** rock, the pressure variation (Δp) with time (t) is described by the diffusion equation assuming the flow is single phase, that Darcy's law holds, and the fluid is slightly compressible. In radial coordinates the diffusion equation can be written as

$$\frac{\kappa_i}{\mu}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Delta p}{\partial r}\right) = \phi_i c_i \frac{\partial \Delta p}{\partial t}, \tag{1}$$

where μ is the fluid viscosity, κ_i is the rock's permeability and ϕ_i is the porosity, with the subscript i indicating these properties are measured on an elastically isotropic rock. c_i is the total compressibility of the isotropic pore/fluid system which can be expanded as:

$$c_i = \frac{1}{K_F} + c_{pp,i}, \tag{2}$$

where K_F is the fluid bulk modulus, and $c_{pp,i}$ is the pore space compressibility of the isotropic pores.

The first term in equation (2) is the fluid's contribution to the total compressibility of the pore/fluid system. The second term ($c_{pp,i}$) reflects the excess pore fluid that can be stored in the pore space (V_p) due to an increase in the pore pressure (p_p) at a constant confining pressure (p_c):

$$c_{pp,i} = \frac{1}{V_p}\left(\frac{\partial V_p}{\partial p_p}\right)_{p_c}. \tag{3}$$

The product of the porosity and the total pore/fluid system compressibility ($\phi_i c_i$) is defined as the storage capacity of the reservoir rock, which can be estimated from the well test data analysis based on the solution of equation (1) (see Appendix A). Since the storage capacity has information about porosity (ϕ_i), fluid compressibility ($1/K_F$) and pore space compressibility($c_{pp,i}$), I show below that it can be used to analyze the changes in seismic wave velocity through the rock with different fluid saturations.

Storage capacity and the fluid substitution problem

One of the objectives of rock physics, is to predict the changes in the velocity of a seismic wave due to changes in the fluid saturation of the rock. This fluid substitution problem can be addressed by finding equations that predict the velocities of the wave propagating through a saturated rock (saturated rock velocities) from the velocities of the wave propagating through the dry rock (dry rock velocities).

Gassman's (1951) equations predict the saturated rock velocities from the dry rock velocities under the assumption that the rock is isotropic, monomineralic and that the pore pressure is equilibrated throughout the pore space. Therefore, Gassman's equations are applicable to the **ISP** rock model defined above, for which the storage capacity of the rock ($\phi_i c_i$) can be measured from pressure-transient analysis. For an isotropic rock, the wave velocities are determined by the bulk (K) and shear (G) moduli and the bulk rock density (ρ). Gassman's (1951) equations for the change in K and G are

$$\frac{1}{K_s} = \frac{1}{K_{d,i}} - \frac{\left(\frac{1}{K_{d,i}} - \frac{1}{K_m}\right)^2}{\left(\frac{1}{K_F} - \frac{1}{K_m}\right)\phi_i + \left(\frac{1}{K_{d,i}} - \frac{1}{K_m}\right)}, \tag{4}$$

$$G_s = G_d, \tag{5}$$

where K_m is the bulk modulus of the mineral material, the "d, i" subscripts stand for measurement done on the dry, isotropic rock and the "s" subscript stands for a measurement done on a rock saturated with a fluid with bulk modulus K_F.

One of the problems in applying equations (4) and (5) to seismic data is that the prediction of K_s depends on the parameters K_F, ϕ_i, $K_{d,i}$ and K_m, which are usually estimated from well logs and cores at a much smaller scale than seismic scale. However, the scale of measurement of the storage capacity estimated from pressure-transient tests is determined by the drainage radius which can be of the order of tens to hundreds of meters. Therefore, the introduction of storage capacity into Gassmann's equation could provide estimates of K_s that are more consistent with the seismic scale of measurement.

In monomineralic, porous rocks, the existence of two different volumes (bulk and pore volumes) and two different pressures (pore and confining pressures) results in four rock compressibilities (Zimmerman, 1991). Each of these compressibilities relates changes in either the bulk volume (V_b) or pore volume (V_p) to changes in the confining pressure (p_c) or pore pressure (p_p), and are defined as

$$c_{bc} = \frac{-1}{V_b}\left(\frac{\partial V_b}{\partial p_c}\right)_{p_p} \equiv \textit{dry bulk compressibility},$$

$$c_{bp} = \frac{1}{V_b}\left(\frac{\partial V_b}{\partial p_p}\right)_{p_c} \equiv \textit{subsidence compressibility},$$

$$c_{pc} = \frac{-1}{V_p}\left(\frac{\partial V_p}{\partial p_c}\right)_{p_p} \equiv \textit{compaction compressibility},$$

$$c_{pp} = \frac{1}{V_p}\left(\frac{\partial V_p}{\partial p_p}\right)_{p_c} \equiv \textit{pore compressibility}. \tag{6}$$

The pore compressibility c_{pp} is the second term in the expression of the total pore/fluid system compressibility in equation (2) and c_{bc} is the inverse of the dry rock bulk modulus ($K_{d,i}$). Appendix B shows three relations among the compressibilities in equation (6) that allow c_{pp} to be expressed as a function of $K_{d,i}$, ϕ_i and K_m:

$$c_{pp} = \left(\frac{1}{K_{d,i}} - \frac{1+\phi_i}{K_m}\right)\frac{1}{\phi_i}. \tag{7}$$

Substituting equation (7) into the definition of c_i in equation (2), the storage capacity of the reservoir is

$$\phi_i c_i = \left(\frac{1}{K_F} - \frac{1}{K_m}\right)\phi_i + \left(\frac{1}{K_{d,i}} - \frac{1}{K_m}\right). \tag{8}$$

Comparison of equations (8) and (4) shows that the denominator on the right side of equation (4) is the storage capacity of the rock. Therefore, the storage capacity measured from the well test experiment can be used to rewrite Gassman's equation (4) as

$$\frac{1}{K_s} = \frac{1}{K_{d,i}} - \frac{\left(\frac{1}{K_{d,i}} - \frac{1}{K_m}\right)^2}{\phi_i c_i}. \tag{9}$$

This equation highlights the benefit of adding the information provided by the pressure-transient analysis. The advantage of expression (9) over the Gassmann's equation is that *no assumptions* are needed about the values of the fluid bulk modulus or formation porosity to calculate K_s because they are included in the storage capacity, $\phi_i c_i$. However, it is still necessary to have estimates of the mineral and dry rock bulk moduli.

From an estimate of the bulk modulus of the rock saturated with a fluid having bulk modulus K_{F1} (e.g. from estimates of P-, S-wave velocities and density), equation (9) can be used to predict the change in the bulk modulus caused by substitution of a second fluid (K_{F2}). This requires a time-lapse well test experiment with one pressure-transient test done with the original fluid and a second after the fluid substitution (e.g. after production or a fluid injection process). In this scenario, the storage capacity estimated from the first test $[(\phi_i c_i)_1]$ would be used with the estimate of saturated bulk modulus (K_{s1}) to calculate ($K_{d,i}$) according to

$$\frac{1}{K_{d,i}} = \frac{1}{K_m} + \frac{(\phi_i c_i)_1}{2}\left[1 - \sqrt{1 - \frac{4\left(\frac{1}{K_{s1}} - \frac{1}{K_m}\right)}{(\phi_i c_i)_1}}\right]. \tag{10}$$

Then, the estimated $K_{d,i}$ can be substituted in equation (9), with the storage capacity estimated from the second pressure-transient test $[(\phi_i c_i)_2]$, to predict the new saturated rock bulk modulus (K_{s2}).

An important observation is that equation (9) predicts that the change in the saturated rock compressibility ($\frac{1}{K_s}$) is proportional to the change in the inverse of the storage capacity i.e.

$$\Delta\left(\frac{1}{K_s}\right)_{21} = -\left(\frac{1}{K_{d,i}} - \frac{1}{K_m}\right)^2 \Delta\left(\frac{1}{\phi_i c_i}\right)_{21}, \qquad (11)$$

where

$$\Delta\left(\frac{1}{K_s}\right)_{21} = \frac{1}{K_{s2}} - \frac{1}{K_{s1}},$$

$$\Delta\left(\frac{1}{\phi_i c_i}\right)_{21} = \frac{1}{(\phi_i c_i)_2} - \frac{1}{(\phi_i c_i)_1}.$$

If seismic and pressure-transient experiments are performed and then repeated after a fluid substitution process, the proportionality shown in equation (11) will hold if the fluid substitution process does not change $(\frac{1}{K_{d,i}} - \frac{1}{K_m})$. Appendix B shows that

$$c_{pc,i}\phi_i = \frac{1}{K_{d,i}} - \frac{1}{K_m}, \qquad (12)$$

where $c_{pc,i}$ is the compaction compressibility defined in equation (6). Therefore, the proportionality will hold if pressure changes in the reservoir during the well test experiment are not high enough to alter the porosity or compaction compressibility of the reservoir. This should hold true for most well compacted reservoir rocks.

PART II: ANISOTROPIC DOUBLE POROSITY ROCK

The second model is the **anisotropic, double porosity (ADP)** rock, which is composed of an isotropic, homogeneous mineral phase and two porous phases. The two different pore types will be referred to as the **isotropic pores** and the **fracture pores**, respectively. The first type of pores are deemed **"isotropic"** because their inclusion in the mineral phase does not render the rock anisotropic. These may be thought of as pores of arbitrary shape that are randomly oriented as in the case of the **ISP** model discussed before. On the other hand, the fractures are assumed to be low aspect ratio pores that have preferential orientations and make the rock elastically anisotropic. It will also be assumed that the internal structure of the **ADP** rock can be described as a group of isotropic blocks, where the isotropic pores reside, that are separated by the network of through-going fractures as shown in Figure 2.

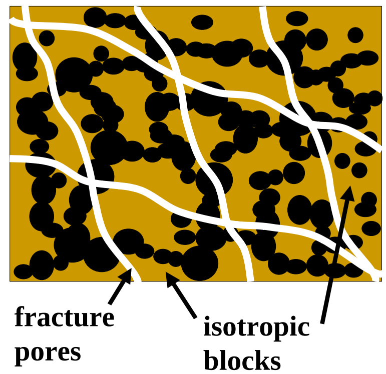

Figure 2. Schematic representation of the anisotropic, double porosity rock. The rock is composed of elastically isotropic blocks separated by fractures.

With regard to flow properties, the isotropic pores have most of the storage capacity of the rock but have small permeability. Flow can occur between isotropic pores and fractures but flow into the wellbore occurs only through the fracture network. In well test analysis this is called the double porosity model (Barrenblatt et al., 1960; Warren and Root, 1963).

As in the single porosity rock, the link between pressure-transient analysis and the rock's elasticity is in the storage capacity. However, in this model the parameter that can be measured is the ratio of the fracture system storage capacity to the total storage capacity of the rock (storage capacity ratio) which is defined as

$$\omega = \frac{\phi_f c_f}{\phi_f c_f + \phi_i c_i}, \qquad (13)$$

where $\phi_f c_f$ is the storage capacity of the fracture pores and $\phi_f c_f + \phi_i c_i$ is the total storage capacity of the rock.

Measuring the storage capacity ratio, ω

In the **ADP** rock, the pressure variation (Δp) with time (t) is also described by the diffusion equation. However, the presence of two pore types with different storage and flow capacities requires the definition of two differential equations (Da Prat, 1990). The first differential equation describes the flow through the fracture network into the wellbore, which can be written in radial coordinates as

$$\frac{\kappa_f}{\mu}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Delta p_f}{\partial r}\right) = \phi_f c_f \frac{\partial \Delta p_f}{\partial t} + \phi_i c_i \frac{\partial \Delta p_i}{\partial t}, \qquad (14)$$

where μ is the fluid viscosity, κ and ϕ are the permeability and porosity, respectively, with the subscript "f" indicating fracture pores and the subscript "i" indicating isotropic pores. c_f and c_i are the compressibilities of the fracture and isotropic pore systems, respectively.

Two pressure variations are indicated in equation (14), one in the fractured pore system (Δp_f) and one in the isotropic pore system (Δp_i). Since flow into the wellbore occurs only through the fracture network, the second term on the right side of equation (14) represents the volume of fluid flowing from the isotropic pores into the fracture system. The rate of flow into the fracture system is determined by the pressure differential between isotropic and fracture pores ($\Delta p_f - \Delta p_i$) and the permeability of the isotropic pores (κ_i). Therefore, the second differential equation is

$$\phi_i c_i \frac{\partial \Delta p_i}{\partial t} = \frac{\kappa_i}{\mu} \frac{(\Delta p_f - \Delta p_i)}{L^2}, \tag{15}$$

where L is the characteristic length of the isotropic rock block.

From well test data analysis based on the solution of differential equations (14) and (15), it is possible to estimate the storage capacity ratio (see Appendix C). The ω parameter includes information about the fracture porosity and fracture pore space compressibility that links well test experiments with elastic parameters that can be estimated from seismic data.

ω parameter and normal fracture compliances

When fractures are introduced into an isotropic porous rock, the overall compressibility of the rock is increased due to the excess compliance associated with the fracture system (Schoenberg and Sayers, 1989). Therefore, the compressibility of the fractured rock is

$$\frac{1}{K_{d,(i+f)}} = Z_{Nf} + \frac{1}{K_{d,i}}, \tag{16}$$

where Z_{Nf} is the normal compliance of the fracture system, $K_{d,i}$ is the bulk modulus of the isotropic part of the rock and the subscript "$(i+f)$" indicates that the influence of both isotropic and fracture pores is accounted for in the overall rock compressibility.

If the storage capacity ratio can be expressed in terms of the dry rock compressibility (16), it will also be a function of the normal compliance of the fracture system, Z_{Nf}. Since estimates of Z_{Nf} can be obtained from seismic data for some types of fractured rocks (Bakulin et al., 2000), having ω as a function of the Z_{Nf} implies it may be possible to obtain estimates of the normal fracture compliance (Z_{Nf}) from pressure-transient analysis or estimates of storage capacity ratio (ω) from seismic data analysis.

The derivations presented in Appendix D show that the ω parameter can be written as

$$\omega = \frac{\left(\frac{1}{K_F} - \frac{1}{K_m}\right)\phi_f + \left(\frac{1}{K_{d,(i+f)}} - \frac{1}{K_{d,i}}\right)}{\left(\frac{1}{K_F} - \frac{1}{K_m}\right)\phi_T + \left(\frac{1}{K_{d,(i+f)}} - \frac{1}{K_m}\right)}, \tag{17}$$

where $\phi_T = \phi_i + \phi_f$. Inserting equations (12) and (16) into equation (17), ω is expressed as a function of the normal fracture compliance Z_{Nf}:

$$\omega = \frac{\left(1 - \frac{K_F}{K_m}\right)\phi_f + K_F Z_{Nf}}{\left(1 - \frac{K_F}{K_m}\right)\phi_T + K_F Z_{Nf} + K_F c_{pc,i}\phi_i}. \tag{18}$$

The disadvantage of expression (18) is that it has too many parameters, few of which are known in practice. However, it is possible to find approximations that will simplify the expression for ω in the limiting cases of very incompressible fluids ($K_F \to 3\ GPa$) or in the case of very compressible fluids ($K_F \to 0\ GPa$).

If the reservoir fluid is a gas at low effective pressures ($K_F \to 0\ GPa$), equation (18) yields

$$\omega \to \frac{\phi_f}{\phi_T}. \tag{19}$$

If the reservoir fluid is brine ($K_F \to 3\ GPa$) then a good approximation to ω is

$$\omega \approx \frac{K_F Z_{Nf}}{\phi_T + K_F Z_{Nf}}. \tag{20}$$

The advantage of expressions (19) and (20) is that they can be used to predict the values of ϕ_f or Z_{Nf} from estimates of ω, provided that ϕ_T and K_F are known. Alternatively, equation (20) can be used to estimate ω from seismically derived values of Z_{Nf}. Since ω is one of at least three parameters that must be estimated from pressure-transient analysis (Appendix C), seismically derived values of ω can be helpful to constrain the parameter inversions done by the well test engineer.

Information about the fracture density of the rock (D_f) can be extracted from equation (20) assuming a specific micro-structural description of the fractures. If the fracture pores behave elastically as penny-shaped cracks, Z_{Nf} is (Bakulin et al., 2000)

$$Z_{Nf} = \frac{A_N D_f}{M_i (1 - A_N D_f)}, \tag{21}$$

where

$$A_N = \frac{4}{3\frac{V_{Pi}}{V_{Si}}\left(1 - \frac{V_{Pi}}{V_{Si}}\right)}, \tag{22}$$

M_i is the P-wave modulus of the isotropic background rock, and V_{Pi} and V_{Si} are the P- and S-wave velocities of the isotropic background rock. Furthermore, the fracture density (D_f) is a function of the fracture porosity and the fracture aspect ratio (α_f):

$$D_f = \frac{3\phi_f}{4\pi\alpha_f}. \tag{23}$$

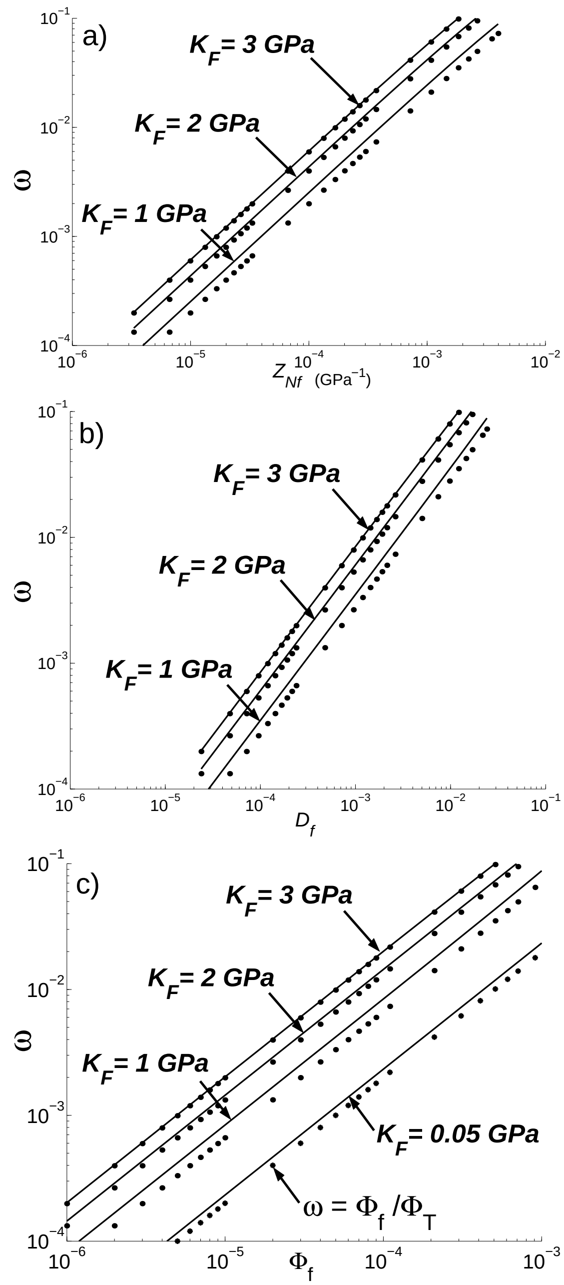

Figure 3. ω plots calculated for different fluid bulk moduli K_F and $\phi_T = 0.05$. Solid lines indicate the exact result from equation (18) and dots show the prediction by the approximation (20). (a) ω vs. fracture compliance, Z_{Nf} (b) ω vs. fracture density, D_f, with the fracture compliance calculated from equations (21) and (22) assuming a calcite mineral matrix. (c) ω vs. fracture porosity, ϕ_f, calculated from equations (21)-(23) assuming a calcite mineral matrix and a crack aspect ratio of 0.01. Also indicated is the continuous line showing how the approximation $\omega \approx \frac{\phi_f}{\phi_T}$ is good for low fluid bulk modulus values ($K_F = 0.05GPa$).

Note that equations (20)-(23), make it possible to estimate the fracture density and fracture porosity from ω. However, this requires strong assumptions that approximate the geometry of the fractures as smooth ellipsoidal cracks.

Figures 3a-3c show the exact (solid lines) and approximate (dots) results of ω versus fracture compliance, fracture density and fracture porosity for a rock with a total porosity $\phi_T = 0.05$. The exact curves in Figure 3b are calculated for a calcite mineral and spherical isotropic pores, and a fracture aspect ratio $\alpha_f = 0.01$ is used in Figure 3c.

The approximate equation (20) becomes more accurate for larger values of the fluid bulk modulus in all three plots. Figure 3c shows that the approximation (19) is better for fluids with bulk modulus less than 0.05 GPa, indicating that well test analysis in gas-producing fractured formations could give estimates of the ratio of fracture porosity to total porosity.

In practice, estimating the fracture compliance (Z_{Nf}) in a water-producing well involves the fewest assumptions; ω is provided by the well-test analysis, K_F can be measured on fluid samples, and ϕ_T can be estimated from well log porosity measurements. Calculating fracture density requires estimates of M_i, V_{Pi} and V_{Si} of the background isotropic rock. Finally, estimates of fracture porosity will require knowledge of all of the above parameters plus the fracture aspect ratio, except for gas wells, where only an estimate of ϕ_T is necessary.

Analysis of Weyburn field data

Weyburn field is a carbonate reservoir consisting of a 30 m interval of dolomite and limestone. There is extensive proof from production, core, and borehole data that the reservoir is fractured. Figure 4 shows pressure data acquired at one of the wells during a pressure build-up test. Since the well is mainly a water producer (oil/water ratio = 2 %), it is ideal for applying equation (20) to calculate the normal fracture compliance from the ω estimate.

Data preparation, quality control, and analysis was done using the commercial software *Saphire*. The PVT data were taken from the model provided by the field operator, and all the fluids' information was taken from data acquired in a well 200 m away from the test well.

The inversion algorithm gives estimates of wellbore storage (C), skin parameter (S), initial pressure (p_{ini}), flow capacity ($\kappa_f h$), and ω and λ parameters (See Appendices A and C for definitions). Figure 5 shows four model curves from four inversion trials with widely different starting set of parameters. Table 1 summarizes the inversion results for four trials and shows that the well-

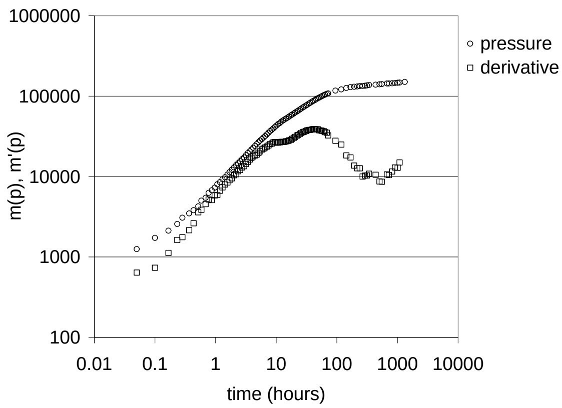

Figure 4. Pressure data and derivative vs. time for the Weyburn field well. Since the well is not producing a single fluid phase (98% water, 2% oil) the axis is expressed in terms of pseudo pressure (**m(p)**) in units of psi^2 and derivative of pseudo pressure (**m'(p)**).

Table 1. Inversion results for four trial examples.

	Fit 1	Fit 2	Fit 3	Fit 4
$C\ (m^3/Kpa)$	0.040	0.039	0.043	0.037
S	-8.636	-8.944	-8.719	-12.109
$p_{ini}\ (Pa)$	23603	24800	24000	29000
$\kappa_f h\ (md - ft)$	274	177	244	41
ω	0.044	0.041	0.043	0.037
$\lambda x10^{-8}$	8	0.1	9	0.005

bore storage (C) and storage capacity ratio (ω) are the most stable estimates.

Based on the average of the storage ratios presented in Table 1 ($\omega = 0.042$), the compressibility of a fluid mixture based on the well's water cut (98 % water, 2 % oil at 15 GPa differential pressure) and an average formation porosity $\phi_T = 0.2$, the normal fracture compliance estimated from equation (20) is $Z_{Nf} \approx 0.003GPa^{-1}$. Under the assumption of penny shaped fractures and limestone isotropic background, the estimated fracture compliance corresponds to a fracture density of $D_f \approx 0.03$. For an average aspect ratio $\alpha_f = 3x10^{-4}$ of fractures observed in Weyburn core (Bunge, 2000), the predicted fracture porosity is $\phi_f \approx 4x10^{-5}$. However, the fracture porosity estimated from Weyburn core averages $\phi_{f,core} \approx 3x10^{-4}$, which is an order of magnitude larger than the well test estimate.

Larger values of fracture porosity are expected from the core measurements due to the reduction in the confining stress that occurs when the core is extracted from the subsurface. Therefore, the porosity derived from well test analysis can be considered an *in situ* estimate that gives a lower bound to the possible values of fracture porosity.

The estimate of the fracture compressibility, $Z_{Nf} \approx 0.003GPa^{-1}$, is likely more reliable than the porosity estimate because of the smaller number of assumptions required for its calculation. In this data set, the stability of the inverted ω parameter facilitates obtaining an estimate of the fracture compressibility, Z_{Nf}. Alternatively, where the storage capacity ratio is hard to obtain, Z_{Nf} estimates from seismic data could be used to constrain the inversion of the ω parameter.

Discussion and Conclusions

In elastically isotropic reservoirs the storage capacity estimated from pressure-transient data, includes information of fluid compressibility and rock porosity averaged over the radius of investigation of the well test. This information can be used directly in Gassmann's equation to predict the changes in the rock's bulk compressibility with saturation.

In fractured anisotropic rocks, the storage capacity ratio (ω) provides information of the fracture compliance, fracture density and fracture porosity. In gas-producing wells in which the gas at reservoir conditions has a bulk modulus smaller than $0.05GPa$, the ω parameter can be used as an approximate measure of the fracture porosity to total porosity ratio ($\omega \approx \frac{\phi_f}{\phi_T}$). If the unfractured porosity (ϕ_i) is measured from well log tools, the fracture porosity can be approximated from estimates of ω.

If the well test experiment is done on a water saturated section of the fractured reservoir (high fluid bulk modulus), the storage capacity ratio can be approximated by a simple equation that is a function of the normal compliance of the fracture system (Z_{Nf}). In this case ω can be estimated from seismically derived values of the normal fracture compliances and vice versa. If a specific model of fractures is assumed, such as an ellipsoidal crack model, the analysis can be taken further to predict the fracture density from the storage capacity ratio.

Even in cases where there is no idealized model that can quantitatively describe the fracture compliances as a function of the fracture density, these two will always be related qualitatively. In other words, a large value of the normal fracture compliance has to be associated

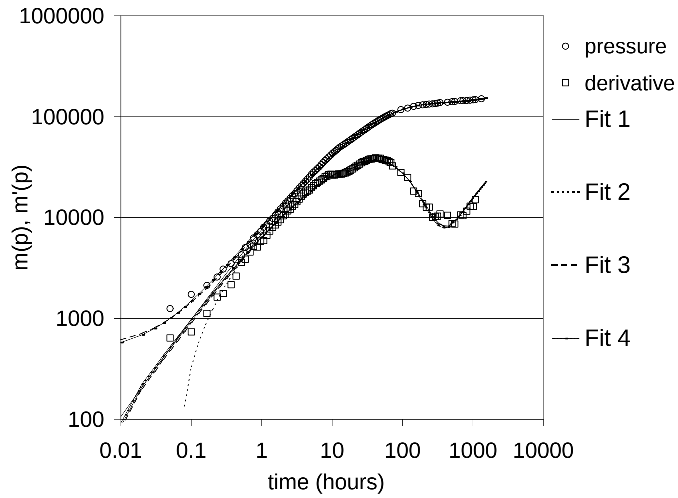

Figure 5. Best fit models for the four parameter inversions shown in Table 1

to a large value of the fracture density. Since the exact equation for ω indicates that large values of fracture compliance correspond to large values of fracture density, we must conclude that the same qualitative relation will exist between ω and the fracture density. Therefore, if seismic and pressure-transient data are analyzed simultaneously in a reservoir that behaves as an anisotropic, double porosity rock, it should be possible to quantitatively correlate the variation in storage capacity ratio with variation of the normal fracture compliance.

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Table 2. Nomenclature

C	wellbore storage
c	compressibility
D_f	fracture density
ϕ	porosity
G	shear modulus
h	formation thickness
K	bulk modulus
κ	permeability
λ	well test λ parameter
M	P-wave modulus
μ	fluid viscosity
p	pressure
q	flow rate
r	radial distance to well
r_w	wellbore radius
ρ	density
S	skin factor
t	time
V_p	pore volume
V_P	P-wave velocity
V_S	S-wave velocity
ω	storage capacity ratio
Z_N	normal fracture system compliance
Subscripts	
d	dry measurement
f	fracture
F	fluid
i	isotropic
s	sat
m	mineral

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APPENDIX A: Measuring storage capacity in the ISP rock

The solution to equation (1) will depend on the inner and outer boundary conditions of the system. If the pressure disturbance does not reach the reservoir boundaries during the time used for data analysis, the outer boundary condition corresponds to an infinite reservoir (transient flow solution). The inner boundary condition depends on the borehole damage or skin effect, and borehole storage. The skin effect reflects a small region around the wellbore with altered permeability values due to damage done during drilling and completion. The borehole storage effect results from fluids that are stored in the wellbore volume and are produced before the formation fluids at the beginning of the well test (Raghavan, 1993).

The late-time response of the infinite reservoir solution to equation (1) is

$$p_D(t_D) = \frac{1}{2} ln \left(\frac{4t_D}{e^\gamma} \right) + S, \tag{A1}$$

where S is the skin factor, γ is Euler's number and p_D and t_D are dimensionless pressure and time, defined as

$$p_D = \frac{2\pi\kappa_I h}{q\mu} \Delta p,$$

$$t_D = \frac{\kappa_I}{\phi_I c_{TI} \mu r_w^2} \Delta t, \tag{A2}$$

where r_w is the wellbore radius, h is the formation thickness and Δp and Δt are the measured pressure change and time.

From equation (A1), the combination of parameters $\phi_i c_i h$ and $\kappa_i h$ can be inverted from the data through a type curve-matching procedure (Lee, 1982). If the thickness of the producing interval is known, the reservoir storage capacity ($\phi_i c_i$) and reservoir permeability (κ_i) are obtained.

APPENDIX B: Storage capacity as a function of $K_{d,i}$, K_{Fl}, K_m and ϕ_i

Zimmerman (1991) shows that, for monomineralic rocks with isotropic mineral material, three equations relate the four compressibilities defined in equation (6):

$$c_{bp} = \frac{1}{K_d} - \frac{1}{K_m}, \tag{B1}$$

$$c_{pp} = c_{pc} - \frac{1}{K_m}, \tag{B2}$$

$$c_{bp} = c_{pc}\phi_i. \tag{B3}$$

Note that even though the *mineral* material must be isotropic for these equations to be valid, the *porous* rock may be anisotropic with arbitrary symmetry. Therefore, the subscript "i" that indicates isotropic pores and subscript "$(i+f)$" that indicates isotropic and fracture pores are omitted.

Substituting equation (B3) into equation (B1) produces

$$c_{pc}\phi_i = \frac{1}{K_d} - \frac{1}{K_m},$$

which is equation (12). If the previous expression is substituted in equation (B2), c_{pp} can be expressed as

$$c_{pp} = \left(\frac{1}{K_d} - \frac{1+\phi}{K_m} \right) \frac{1}{\phi},$$

which is the expression presented in equations (7) and (D2).

APPENDIX C: Transient-pressure analysis in the ADP rock

The differential equations (14) and (15) are solved in Laplace space after defining dimensionless pressures and times as in equations (A2) (Da Prat, 1990). The pressure response of the double-porosity rock presents three regimes: early, transitional and late, which are shown in Figure C1. In the early period the pressure solution is

$$P(t)_{early} = \frac{q\mu^{1/2}}{hr_w\kappa_f^{1/2}\pi^{3/2}} \sqrt{\frac{t}{\phi_f c_f}}, \tag{C1}$$

where q is the flow rate, μ is the fluid viscosity, h is the formation thickens and r_w is the wellbore radius. Note that at early times the pressure response is dominated by the fracture storage capacity which means that the fracture system does not "sense" the presence of the isotropic porosity. In the late period the pressure solution is

$$P(t)_{late} = \frac{q\mu}{4\pi h r_w \kappa_f} ln \left(\frac{4\kappa_f t}{\mu r_w^2(\phi_f c_f + \phi_i c_i)} \right) - \gamma, \tag{C2}$$

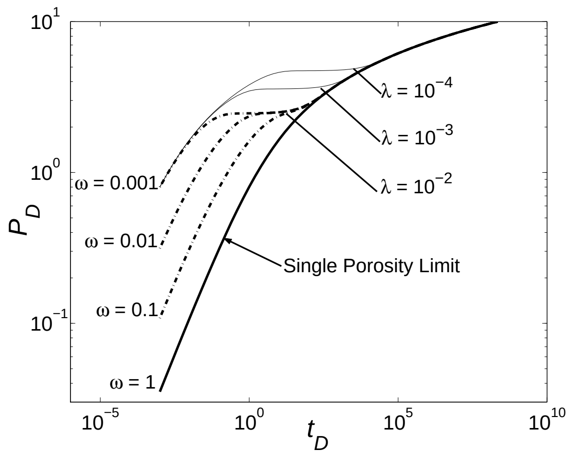

Figure C1. Double porosity pressure response indicating the three regimes and the influence of λ in determining the transition between the early and late periods.

where γ is Euler's number. In the late-time period the response is identical to that of a single-porosity rock with a storage capacity equal to $(\phi_f c_f + \phi_i c_i)$, which means that the fracture system "senses" the presence of the isotropic porosity.

The transition between the fracture and total rock system regimes is determined by the parameter λ defined as

$$\lambda = \frac{\kappa_i}{\kappa_f} \frac{r_w^2}{L}, \tag{C3}$$

where L is the characteristic length of an isotropic rock block. If the permeability of the isotropic pores is small, the transition will occur later because it will take longer to transfer fluids from the low-permeability isotropic rock into the fracture system.

The value of the ω parameter is always between zero and one. If the rock has no isotropic porosity (e.g. anisotropic, fractured granite), $\omega = 1$ and the pressure response will be identical to that of a single porosity rock at all times. If ω is small but non-zero, the λ parameter determines at which time the pressure response converges to the single porosity limit (see Figure C1). If the rock has no fracture porosity (e.g. unfractured, tight carbonate) $\omega = 0$. In this limit, the rock model becomes impermeable because of the lack of fractures, and no pressure drop occurs.

APPENDIX D: Relating ω to the fractured rock compressibility

To find an equation that relates ω to the normal fracture compressibility (Z_{Nf}) it is necessary to expand the numerator and denominator of equation (13). The denominator is the total rock-storage capacity, which can be written as

$$(\phi_f c_f + \phi_i c_i) = \phi_T \left(\frac{1}{K_F} + c_{pp,(i+f)} \right), \tag{D1}$$

where $\phi_T = \phi_i + \phi_f$ and $c_{pp,(i+f)}$ is the total pore space compressibility defined in equation (6), which includes fracture pores and isotropic pores. Note that the only difference between the compressibility defined for the single porosity rock $(c_{pp,i})$ and $c_{pp,(i+f)}$ is that the latter must take into account the fracture pores.

From the relations between compressibilities presented in Appendix B, $c_{pp,(i+f)}$ can be expressed as a function of the dry rock compressibility, mineral compressibility, and total porosity as

$$c_{pp,(i+f)} = \left(\frac{1}{K_{d,(i+f)}} - \frac{1 + \phi_T}{K_m} \right) \frac{1}{\phi_T}, \tag{D2}$$

which is similar to equation (7) except that $K_{d,(i+f)}$ is the dry bulk modulus of a fractured, anisotropic rock.

Substituting (D2) into (D1), the denominator of the ω parameter is

$$(\phi_f c_f + \phi_i c_i) = \left(\frac{1}{K_F} - \frac{1}{K_m} \right) \phi_T + \left(\frac{1}{K_{d,(i+f)}} - \frac{1}{K_m} \right). \tag{D3}$$

Since the isotropic pores of the **ADP** model are identical to the pores in the **ISP** rock, equation (8) can be substituted into equation (D3) to solve for $\phi_f c_f$ as

$$\phi_f c_f = \left(\frac{1}{K_F} - \frac{1}{K_m} \right) \phi_f + \left(\frac{1}{K_{d,(i+f)}} - \frac{1}{K_{d,i}} \right). \tag{D4}$$

Finally, from equations (D3) and (D4) we can write the ω parameter as

$$\omega = \frac{\left(\frac{1}{K_F} - \frac{1}{K_m} \right) \phi_f + \left(\frac{1}{K_{d,(i+f)}} - \frac{1}{K_{d,i}} \right)}{\left(\frac{1}{K_F} - \frac{1}{K_m} \right) \phi_T + \left(\frac{1}{K_{d,(i+f)}} - \frac{1}{K_m} \right)}. \tag{D5}$$