CWP-487

A formalism for the suppression of multiple diffractions

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ABSTRACT
Diffracted multiples can be a problem in marine surveys since these waves appear in seismic data as a curtain of noise that obscures primaries. In this work I present a theory to suppress diffracted multiples that is based on a two-step process. The first step consists of the estimation of the scatterers near the sea-bottom. In the second step the reflections from the free surface are removed. There are reasons to believe that theoretical advances in the field of multiple suppression, as presented here, are of limited value unless limitations in the data acquisition can be overcome.

Key words: volcano monitoring, natural hazards, deconvolution

1 INTRODUCTION

Diffracted multiples are a major problem in areas with a rough sea-bottom, such as the Orme-Lange field, because the diffracted multiples obscure primaries that are the main target of the survey. The travel time curves of the diffracted multiples are steeper that those of the diffracted primaries, often to the extent that the diffracted multiples are aliased and show up in seismic data as a curtain of noise. Standard multiple elimination techniques cannot cope with these incoherent multiple diffractions, especially when the multiple elimination is carried out in the wavenumber domain, e.g. Verschuur et al. (1992).

In this paper the theory for the suppression of diffracted multiples is formulated as a two-step process. In the first step the scatterers near the sea-bottom are estimated, in the second step the reflections from the free water surface is removed (Noah's deconvolution (Riley & Claerbout, 1976)). The first step is based on a data-fitting procedure of the diffraction integral using a time window of the data that primarily contains the primary water-bottom diffractions. The removal of the free surface is formulated using the technique of Wapenaar et al. (1996).

It is questionable, though, if theoretical advances are sufficient to achieve a satisfactory suppression of multiple diffractions. These waves are likely to have bounce points at the free surface that may be far removed from the employed recorders, and the data that are needed to solve the integral equation for the multiple suppression may not have been recorded.

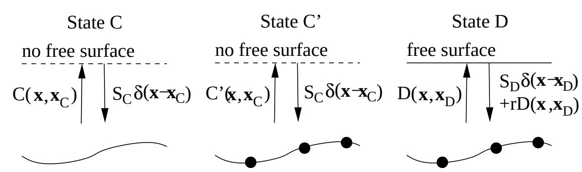

Figure 1. Definition of the corrected state C (without free surface and without scatterers), the intermediate state C' (without free surface but with scatterers), and the data state D (with a free surface and with scatterers). The scatterers are indicated by black circles.

2 COMPARING THREE DIFFERENT WAVE STATES

The basis for the suppression of diffracted and specular multiples forms the comparison of three different wavefields as shown in Figure 1. In this paper I follow Wapenaar et al. (1996), and decompose the wavefield in upcoming and downgoing waves. The pressure field in the water can be written as

$$\mathbf{p} = \begin{pmatrix} p_+ \\ p_- \end{pmatrix} , \tag{1}$$

where p_+ denotes the downgoing waves and p_- the upcoming waves.

Throughout this work the vector $\mathbf{x}$ denotes a two-dimensional vector over a surface. The state D in the right panel of Figure 1 reflects the wavefield in a field experiment; this state is called the *data-state.* Waves are generated by a source at the surface $z = 0$ at a horizontal location $\mathbf{x}_D$ and with source spectrum $S_D(\omega)$. The subsequent analysis is in the frequency domain and the dependence on ω is suppressed . The upcoming wavefield at surface location $\mathbf{x}$ is denoted by $D(\mathbf{x}, \mathbf{x}_D)$. These waves result from specular reflections, diffractions, and their interaction with the free surface. The state C is the *corrected state.* It is the wavefield that would be measured from a source at $\mathbf{x}_C$ with source spectrum S_C in a hypothetical Earth where the water surface is not a free surface and where the ocean bottom generates no diffractions. The ultimate goal of this work is to compute the upcoming waves $p_C^-(\mathbf{x}, \mathbf{x}_C) = C(\mathbf{x}, \mathbf{x}_C)$ because in this wave-state the imprint of the free surface and of the water-bottom diffractions has been removed. In order to formulate the mapping from the D-state to the C-state, I introduce an intermediary C'-state as shown in the middle panel of Figure 1. This is the wave-state that would be recorded if there were no free surface, but the water-bottom still generated diffracted waves.

Let us first consider the upgoing and downgoing waves in the three states. In the C-state and the C'-state, the downgoing waves are caused solely by a source at $\mathbf{x}_C$ that radiates a downgoing wave $\delta(\mathbf{x} - \mathbf{x}_C)S_C$ at the location $\mathbf{x}_C$ just above the ocean surface. The upcoming waves are denoted by $C(\mathbf{x}, \mathbf{x}_C)S_C$; that is, $C(\mathbf{x}, \mathbf{x}_C)$ gives the impulse response. The upgoing and downgoing waves for the C-state are given by

$$\mathbf{p}_C(\mathbf{x}, \mathbf{x}_C) = \begin{pmatrix} \delta(\mathbf{x} - \mathbf{x}_C)S_C \\ C(\mathbf{x}, \mathbf{x}_C)S_C \end{pmatrix} , \qquad (2)$$

with a similar solution for the C'-state. For the D-state, the upcoming wave is given by the recorded data $D(\mathbf{x}, \mathbf{x}_D)$ that are generated by a source at $\mathbf{x}_D$. The downgoing wave is given by the superposition of waves $\delta(\mathbf{x} - \mathbf{x}_D)S_D$ radiated downward by the source, and the waves rp_D^- that are reflected by the free surface, where the reflection coefficient at the free surface is denoted by r. The upgoing and downgoing waves for the D-state are thus given by

$$\mathbf{p}_D(\mathbf{x}, \mathbf{x}_D) = \begin{pmatrix} \delta(\mathbf{x} - \mathbf{x}_D)S_D + rD(\mathbf{x}, \mathbf{x}_D) \\ D(\mathbf{x}, \mathbf{x}_D) \end{pmatrix} . \qquad (3)$$

The ultimate goal is to compute the C-state, given the waves recorded for the D-state. This mapping is formulated here by first relating the C-state to the C'-state, and then relating the C'-state to the D-state. These two mappings are treated in the following subsections.

2.1 Accounting for the water-bottom diffractions

In general, localized scatterers on the water bottom diffract downgoing waves upward, but they also diffract upgoing waves downward, and they influence the transmission through the water bottom. Furthermore, waves may propagate directly from one scatterer to another. In this work I assume that the latter wave propagation phenomena can be ignored; hence, I assume that the scatterers influence the waves only by diffracting downgoing waves upward. Because the C'-state has no free surface, the upcoming waves in the C'-state are given by the upcoming waves in the C-state plus the diffracted waves:

$$C'(\mathbf{x}, \mathbf{x}_C) = C(\mathbf{x}, \mathbf{x}_C) + H(\mathbf{x}, \mathbf{x}_C) . \qquad (4)$$

In this expression $H(\mathbf{x}, \mathbf{x}_C)$ denotes the diffracted waves. These waves can be expressed in an integral over the water bottom:

$$H(\mathbf{x}, \mathbf{x}_C) = \int_{bottom} G_W^-(\mathbf{x}, \mathbf{x}_b)h(\mathbf{x}_b)G_W^+(\mathbf{x}_b, \mathbf{x}_C)d^2\mathbf{x}_b . \qquad (5)$$

In this expression $G_W^{\pm}$ denotes the one-way Green's function for downgoing waves and upgoing waves, respectively, that propagate through the water layer. The diffraction strength at the water bottom is described by $h(\mathbf{x}_b)$. In general this diffraction strength depends on the angles of incidence of the incoming and outgoing waves. When these angles are not too far from the vertical this dependence can be ignored and the diffraction strength depends on the location $\mathbf{x}_b$ at the bottom only.

The integration in expression (5) is over the water bottom of the model without the scatterers. In reality the scatterers may be embedded in the sediments and are therefore located slightly below the water bottom. Also, the precise location of the water bottom may not be known with great accuracy. In that case, one can replace the integration over the water bottom by an integration over a reference surface that is close to the water bottom. A deviation between this reference surface and the location of the scatterer depth over a distance Δz, leads for vertically incident waves to a phase shift $\exp(2ik\Delta z)$; this phase shift can be absorbed in the coefficient $h(\mathbf{x}_b)$. When the direction of wave propagation deviates too much from the vertical, this phase shift is modified by the angle of the incidence of the incoming and outgoing waves. The theory can be extended to accommodate this situation.

In section 3, I outline how the diffraction coefficient $h(\mathbf{x}_b)$ can be estimated. Once this coefficient is known, expressions (4) and (5) can be used to related the C'-state to the C-state.

2.2 Eliminating the free surface

Relating the D-state to the C'-state entails free-surface multiple suppression. A variety of techniques have been developed for this problem (Kennett, 1979; Riley & Claerbout, 1976; Verschuur *et al.*, 1992; Weglein *et al.*, 1998; Wapenaar *et al.*, 2002). In this section I reformulate the theory of Wapenaar et al. (Wapenaar & Grimbergen, 1996; Wapenaar *et al.*, 2002) adapted to the wave states shown in Figure 1. The subsurface medium is the same in the D-state and the C'-state. Following equation (31) of ref. Wapenaar et al. (1996), these states are related by the following representation theorem:

$$\int_{\partial V} \mathbf{p}_{C'}^T \mathbf{N} \mathbf{p}_D n_z d^2 x_H = \int_V \left(\mathbf{p}_{C'}^T \mathbf{N} \mathbf{S}_D + \mathbf{S}_{C'}^T \mathbf{N} \mathbf{p}_D \right) dV \, , \qquad (6)$$

where the vector $\mathbf{S}$ contains the upgoing and downgoing wave components of the source, and where the superscript T denotes the transpose. The matrix $\mathbf{N}$ is given by

$$\mathbf{N} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} . \qquad (7)$$

The bounds ∂V of the volume V in expression (6) are horizontal. For this reason, only the z-component of the outward pointing unit vector appears in the left hand side (6).

In the following I take the upper boundary of the surface just below the sources that excite the wavefield. At that boundary $n_z = -1$. Since the boundary is placed just below the sources, the sources are outside the volume and the right-hand side of (6) vanishes. The lower boundary is placed at infinite depth, so that it does not contribute. Inserting definition (7) and using the representations (2) for the state $C'$and (3) for the state D gives (after division by the source spectrum S_C),

$$\begin{aligned} D(\mathbf{x}_C, \mathbf{x}_D) \quad & -S_D C'(\mathbf{x}_D, \mathbf{x}_C) \\ & -r S_D \int C'(\mathbf{x}, \mathbf{x}_C) D(\mathbf{x}, \mathbf{x}_D) d^2\mathbf{x} = 0 \, . \end{aligned} \qquad (8)$$

By reciprocity, $C'(\mathbf{x}_1, \mathbf{x}_2) = C'(\mathbf{x}_2, \mathbf{x}_1)$. Only the source spectrum S_D appears in this expression, the subscript D in the source spectrum is suppressed in the following. Using this gives, with equation (4),

$$\begin{aligned} & D(\mathbf{x}_C, \mathbf{x}_D) - S\left\{C(\mathbf{x}_C, \mathbf{x}_D) + H(\mathbf{x}_C, \mathbf{x}_D)\right\} \\ & -r S \int \left\{C(\mathbf{x}_C, \mathbf{x}) + H(\mathbf{x}_C, \mathbf{x})\right\} D(\mathbf{x}, \mathbf{x}_D) d^2\mathbf{x} = 0 \, . \end{aligned} \qquad (9)$$

Before analyzing how this expression can be used to suppress diffracted multiples, let us first interpret this expression in terms of scattering diagrams. In a shorthand notation, the previous expression can be written as

$$D = S(C+H) + rS\,(C+H)\,D \, . \qquad (10)$$

The products in this expression should be interpreted according to the notation of equation (9); hence the products entail an integration over the free surface. Iterating equation (10) gives the following expansion:

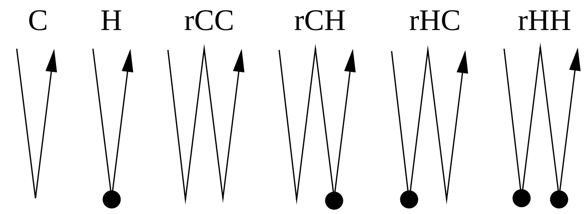

Figure 2. All first- and second-order scattering events in the expansion (11). The scatterers are indicated by black circles.

$$\begin{aligned} D \; & = S(C+H) + rS^2\,(C+H)\,(C+H) \\ & + r^2 S^3\,(C+H)\,(C+H)\,(C+H) + \cdots \\ & = SC + SH + rS^2 CC \\ & + rS^2 CH + rS^2 HC + rS^2 HH + \cdots \end{aligned} \qquad (11)$$

The different terms in the last line are depicted in the scattering diagrams of Figure 2. Each free-surface reflection is associated with a factor r. Each diffraction H is depicted by the scattering by a black circle. The reflections from the subsurface and the specular reflections of the sea bottom that are accounted for by the response C are denoted by a scattering event without a black circle. It follows from Figure 2 that all multiple scattering events where waves are reflected upward by the scatterers are accounted for. Multiple diffractions as well as peg-leg multiple reflections are contained in the series (11). This means that the integral equation (9) accounts for these multiple scattering paths as well.

3 SUPPRESSING THE DIFFRACTED MULTIPLES

In the integral equation (9), the data D have been measured and are thus given, whereas, the source spectrum S and the diffracted waves H are unknown. The goal of the procedure is to obtain the cleaned wavefield C. A parameter count shows that it is impossible to determine S, H, and C from this single equation. By using different time windows, however, these different unknowns can be unraveled.

The estimation of the source spectrum S is exactly the same as in existing procedures for multiple elimination (Verschuur *et al.*, 1992; Ikelle *et al.*, 1998). These references show how the source spectrum can be estimated and how this estimate can be iteratively improved while eliminating successive orders of multiples.

The estimation of C and H can be unraveled by applying a time window to the data that starts after the first water-bottom reflection and ends before the first water-bottom multiple, as shown in Figure 3. In the figure the window is indicated by dashed straight

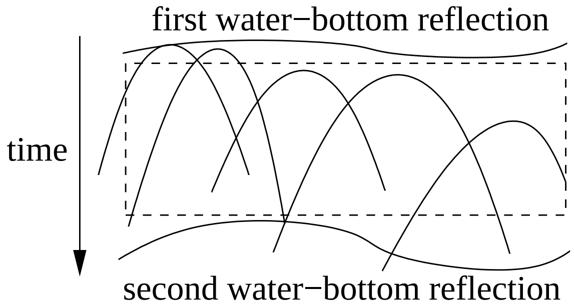

Figure 3. The first and second water-bottom reflection in the time domain with water-bottom diffractions. The time window to be used for estimating the water-bottom diffraction strength $h(\mathbf{x}_b)$ is indicated by the dashed lines.

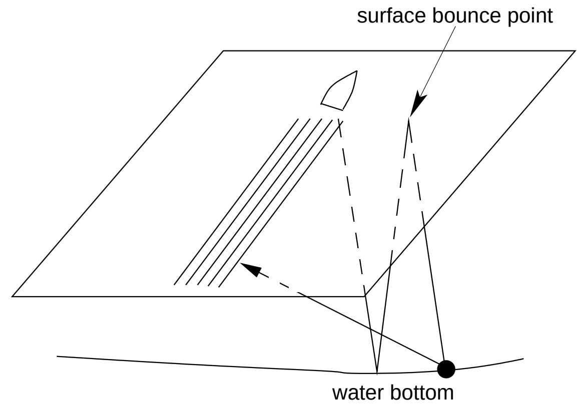

Figure 4. Example of an acquisition geometry where the multiple that diffracts of the solid circle bounces off the sea surface at a location where no data are collected. The employed streamers are indicated by the five lines behind the survey ship.

lines. When the water bottom is sloping, the bounds of the window can follow the first- and second-order water-bottom reflections. Within this time window the wavefield consists of sub sea-bottom primaries and water-bottom diffractions. A least-squares data fit using expression (5) gives the diffraction strength of the water bottom. The sub sea-bottom primaries in this window cannot in general be fitted with the diffraction integral (5) because the travel time curve of a sea-bottom primary differs in general from the travel time curve of a sea-bottom diffraction. This means that the waveform fit using expression (5) effectively gives the diffracted waves H only.

It is important to realize that the goal of this step is not to obtain an accurate estimate of the diffraction strength $h(\mathbf{x}_b)$. Instead, this procedure aims at obtaining an accurate estimate of the diffracted waves H that are consistent with the diffraction integral (5). This procedure is akin to the surface wave elimination method of the group of Herman (Blonk & Herman, 1996; Ernst *et al.*, 1998) where the goal is not to estimate the near-surface scattering coefficients, but instead is to estimate the scattered surface waves.

Once the diffracted waves H are known, the solution of expression (9) constitutes the standard procedure for the elimination of surface multiples by solving this integral equation for C. A number of different techniques could be used:

- Solve the integral equation by brute force. This is probably not a good idea.
- Do a Fourier transform over the horizontal coordinates. In that case the integration reduces to a multiplication. This is the technique used by the Delft group (Berkhout & Verschuur, 1997; Verschuur & Berkhout, 1997). Subsequent orders of multiples are removed by iterating the resulting equation.
- Expand the solution C in terms that depend linearly on the data, quadratic on the data, etc: $C = C_1 + C_2 + \cdots$. This is the approach used by Weglein et al. (2003), which is similar to the iterative solutions used by the Delft group.

4 DISCUSSION

The theory presented here offers in principle a way to divide the removal of diffracted multiples as a two-step process wherein the identification of the scatterers and the removal reflections of the free surface are separate steps. There is a good reason, however, to believe that this formalism will not solve the problem of removing diffracted multiples in acquisition geometries that are common in marine seismic surveys.

The problem is illustrated in Figure 4, which depicts a generic marine survey. Indicated is a diffracted multiple that is generated near the survey vessel, then reflects off the water bottom and the sea-surface, respectively, and is then diffracted by a scatterer near the sea-bottom. Physically, the theory presented here makes is possible to subtract this multiple by combining the information of the primary that propagates from the source to the surface bounce point, with the primary diffraction that propagates from the surface bounce point via the scatterer to the streamers. Mathematically this subtraction process is formulated in the surface integral that appears in expression (9). In this integral the data $D(\mathbf{x}, \mathbf{x}_D)$ term describes the waves that are excited at the source and propagate to the integration point $\mathbf{x}$, which is the bounce point at the free surface.

The problem is that the waves are collected at the streamers only. This means that the data $D(\mathbf{x}, \mathbf{x}_D)$ are not collected at the point $\mathbf{x}$ where they are needed. This drawback is inherent not just in this technique but in any technique that relies on an integration over the whole surface (Weglein *et al.*, 1998), and in formulations for multiple removal where the surface integral is recast as a multiplication in the wavenumber domain using a plane-wave decomposition of the wave field (Verschuur *et al.*, 1992).

This *problem of missing data* is more severe for removal of diffracted multiples than for the removal of multiples from near-horizontal reflectors. In the latter case, and for small reflector dips, the wave paths of multiples don't move far from the vertical recording plane. This means that the surface bounce point of the multiples often lies within, or close to, the streamer array. If needed, an interpolation between the streamers can then be used to carry out the surface integration needed in the multiple elimination. For diffracted multiples the multiples are likely often associated with wave propagation some distance away from the recording plane, and an acquisition geometry based on streamers fails to collect the data necessary to carry out the surface integral needed for the elimination of diffracted multiples. Thus, the elimination of diffracted multiples cannot be achieved with theoretical advancements alone, but rather requires changes in the acquisition geometry in marine surveys.

ACKNOWLEDGEMENTS

I appreciate the discussions with Ray Abma, Bill Dragoset, Ken Larner, and Greg Wimpey.

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