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Exact layer stripping of PP and PS reflections from dipping interfaces in anisotropic media

Pawan Dewangan and Ilya Tsvankin
Center for Wave Phenomena, Department of Geophysics,
Colorado School of Mines, Golden, CO 80401-1887

ABSTRACT

Building accurate interval velocity models is critically important for seismic imaging and AVO (amplitude variation with offset) analysis. Here, we adapt the so-called "PP+PS=SS" method to develop an exact technique for constructing the interval traveltime-offset function in a dipping anisotropic (target) layer beneath a horizontally layered overburden. Whereas the overburden is also supposed to have a horizontal symmetry plane, there are no restrictions on the type of anisotropy in the target layer.

It should be emphasized that the presented algorithm is entirely data-driven and does not require knowledge of the velocity field anywhere in the model. Other important advantages of our method compared to the generalized Dix equations include the ability to handle laterally heterogeneous target layers, long-offset data and mode-converted waves. Numerical tests confirm the high accuracy of the algorithm in computing the interval traveltimes of both PP- and PS-waves in a transversely isotropic layer with a tilted symmetry axis (TTI medium) beneath an anisotropic overburden.

In combination with existing inversion techniques for homogeneous TTI media, the layer stripping of PP and PS data can be used to estimate the interval parameters of TTI formations in such important exploration areas as the Canadian Foothills. Other potential applications of our methodology are in the dip-moveout inversion for the key time-processing parameter η and in the exact computation of the interval long-spread (nonhyperbolic) moveout that provides valuable information for anisotropic velocity analysis.

Key words: reflection moveout, velocity analysis, multicomponent data, mode conversions, anisotropic media

1 INTRODUCTION

Velocity analysis based on reflection moveout is routinely used for estimating subsurface velocity fields and imaging target reflectors. However, reflection traveltime in general and normal-moveout (NMO) velocity in particular represent effective quantities that are influenced by the medium properties along the entire raypath of the reflected wave. Interval parameter estimation for purposes of prestack and poststack migration requires application of layer-stripping (e.g., Dix, 1955; Liu, 1997; Grechka and Tsvankin, 2000; Sarkar and Tsvankin, 2004) or tomographic (e.g., Stork, 1991; Pech et al., 2002a,b) methods. Layer parameters are also needed for the inversion of the AVO (amplitude variation with offset) response, lithology discrimination and fracture detection using seismic data, etc.

In horizontally layered, isotropic media, the NMO velocity of reflected waves is equal to the root-mean-square (rms) of the interval velocities. This simple relationship, first discussed by Dix (1955), makes it possible to obtain the velocity in any layer using only the NMO velocities for the reflections from the top and bottom of this layer. A more general version of the Dix equation for isotropic layered models with dipping interfaces was derived by Shah (1971).

If the medium is transversely isotropic with a vertical symmetry axis (VTI) and laterally homogeneous, NMO velocity is still equal to the rms of the interval NMO velocities (Hake et al., 1984; Tsvankin and Thomsen, 1994). Note, however, that since the interval NMO velocity is distorted by anisotropic parameters, application of the Dix equation does not yield the true interval vertical velocities, which causes mis-ties in time-to-depth conversion (e.g., Tsvankin, 2001). For wide-azimuth, pure-mode (nonconverted) data acquired above a stack of horizontal, arbitrarily anisotropic layers, the exact NMO velocity can be obtained by the Dix-type averaging of the interval NMO ellipses (Grechka et al., 1999). On the whole, as long as the model is laterally homogeneous, the interval NMO velocity or NMO ellipse can still be obtained using just the moveout of the reflection events from the top and bottom of the layer of interest.

As shown by Alkhalifah and Tsvankin (1995) and Tsvankin (2001), the Dix equation remains valid even for reflections from *dipping* interfaces in anisotropic media, if the CMP (common-midpoint) line is confined to a vertical symmetry plane and the overburden is laterally homogeneous. The 3D extension of this result to the NMO ellipses of dipping events is given by Grechka et al. (1999). The main difference between these generalized Dix-type equations and the conventional Dix formula is in the nature of the interval NMO velocity or the interval NMO ellipse. If the reflector is dipping, the NMO velocity in each layer no longer corresponds to any physical interface and has to be computed for a non-horizontal (imaginary) reflector orthogonal to the slowness vector of the zero-offset ray in this layer.

Although this requirement does not pose problems in forward modeling, it causes serious difficulties in interval parameter estimation because recorded reflections from the overburden yield only the NMO velocities for horizontal interfaces. Therefore, velocity-analysis algorithms based on the generalized Dix equations involve interval parameter estimation for the whole overburden. When the medium is anisotropic, the Dix-type layer stripping of NMO velocities or ellipses usually requires both horizontal and dipping events for each depth interval in the overburden (Alkhalifah and Tsvankin, 1995; Grechka and Tsvankin, 2000).

Another principal limitation of the generalized Dix equations and other algorithms operating with NMO velocity is the assumption of symmetric reflection moveout (i.e., the traveltime has to stay the same when the source and receiver are interchanged). While this assumption is always satisfied for pure (non-converted) modes, the moveout of converted PS- or SP-waves is symmetric only in laterally homogeneous media with a horizontal plane of symmetry. Dewangan and Tsvankin (2004a,b) showed that the asymmetric moveout of PS-waves in TI media with a tilted symmetry axis (TTI) provides critically important information for anisotropic parameter estimation. Their inversion algorithm, however, is developed for a single horizontal or dipping TTI layer and cannot be applied to layered media using existing Dix-type formalism.

Finally, the Dix-type equations are derived for NMO velocities that describe only conventional-spread moveout for the maximum offset-to-depth ratio not much larger than unity. Similar averaging expressions for long-offset data involve the quartic moveout coefficient and are limited to horizontally layered, azimuthally isotropic models (e.g., Tsvankin and Thomsen, 1994; Alkhalifah, 1997).

Here, we present a technique for computing the exact interval traveltimes and and offsets of pure-mode and converted waves reflected from a dipping interface overlain by a horizontally layered overburden. Our approach is based on the so-called "PP+PS=SS" method originally designed for generating pure-mode SS reflection data from PP- and PS-waves (Grechka and Tsvankin, 2002b; Grechka and Dewangan, 2003). In contrast to the Dix-type equations, the algorithm does not require any information about the velocity or anisotropic parameters of the overburden and can be applied to long-offset data. The accuracy of the estimated interval moveouts for both PP- and PS-waves is verified by numerical testing on layered transversely isotropic models with the depth-varying tilt of the symmetry axis.

2 LAYER STRIPPING FOR PURE REFLECTION MODES

Let us consider pure (non-converted) reflected waves in an anisotropic medium comprised of a stack of horizontal layers above a dipping reflector. To make the problem two-dimensional, the acquisition line is assumed to be confined to the dip plane of the reflector that should coincide with a vertical symmetry plane in all layers. Therefore, the incidence plane represents a symmetry plane for the model as a whole (Figure 1).

The layer immediately above the dipping interface can be arbitrarily heterogeneous with only one restriction on the type of anisotropy: the dip plane has to be a plane of symmetry. The horizontal layers in the overburden, however, are assumed to be laterally homogeneous with a horizontal symmetry plane. Under these assumptions, any reflection point at the bottom of the overburden (e.g., points T and R in Figure 1) coincides with the common midpoint for the corresponding source-receiver pair, and the traveltimes along the upgoing and downgoing segments of the reflected ray are equal to each other.

If lateral heterogeneity above the reflector is negligible, the NMO velocity from the dipping interface is

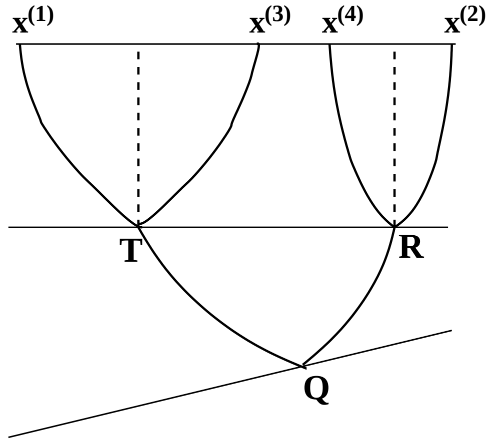

Figure 1. 2D ray diagram of the layer-stripping algorithm designed to find the interval traveltimes and source/receiver coordinates in the layer immediately above the dipping reflector. Points T and R are located at the bottom of the horizontally layered overburden; each layer in the overburden is assumed to have a horizontal symmetry plane. The reflections from the dipping interface ($x^{(1)}TQRx^{(2)}$) and the bottom of the overburden ($x^{(1)}Tx^{(3)}$) share the same downgoing leg ($x^{(1)}T$). The upgoing leg of the dipping event ($Rx^{(2)}$) coincides with a leg of another overburden reflection, $x^{(2)}Rx^{(4)}$.

given by the following Dix-type equation (Alkhalifah and Tsvankin, 1995; Tsvankin, 2001):

$$V_{\rm nmo}^2(N) = \frac{1}{t_0}\sum_{i=1}^{N}\left[V_{\rm nmo}^{(i)}(p)\right]^2\, t_0^{(i)}(p)\,, \tag{1}$$

where N is the number of layers, t_0 is the zero-offset traveltime, $t_0^{(i)}$ is the interval traveltime along the zero-offset ray in layer i, and $V_{\rm nmo}^{(i)}$ is the interval NMO velocity. The difference between equation (1) and the conventional Dix formula is in the meaning of the interval NMO velocity; it has to be computed for an imaginary reflector orthogonal to the slowness vector of the zero-offset ray in each layer. Therefore, determination of the interval NMO velocity in layer N from equation (1) involves computing NMO velocities from non-existent dipping interfaces in the overburden. While this operation is trivial for isotropic media (if the ray parameter is known), in the presence of anisotropy it requires an estimate of the relevant medium parameters in the whole overburden (Alkhalifah and Tsvankin, 1995).

2.1 Layer-stripping algorithm

Here, we show how a variation of the "PP+PS=SS" method (Grechka and Tsvankin, 2002b; Grechka and Dewangan, 2003) can be used to obtain the exact interval traveltime-offset function in the layer immediately above the target dipping reflector. The idea is to combine the reflections from the dipping interface and the bottom of the overburden in such a way that they share the same ray segments in the overburden (Figure 1).

Suppose the dipping event was excited at location $x^{(1)}$ and recorded at location $x^{(2)}$ (or vice versa), and the reflection from the bottom of the overburden was acquired for a sufficiently wide range of source-receiver offsets. Following the methodology of Grechka and Tsvankin (2002b), we form a common-receiver gather of the dipping event at point $x^{(2)}$ and determine the time slope on this gather at point $x^{(1)}$. Since the slowness vector is equal to the gradient of the travelime surface, the estimated time slope coincides with the ray parameter (horizontal slowness) of the reflection $x^{(1)}TQRx^{(2)}$ at the source location $x^{(1)}$. We then use the same algorithm to evaluate the time slopes of the overburden reflections excited at $x^{(1)}$ and recorded at different locations along the line. For a certain receiver location $x^{(3)}$, the time slope (ray parameter) of the overburden reflection from $x^{(1)}$ to $x^{(3)}$ coincides with that of the dipping event,

$$\frac{\partial t^{\rm eff}(x^{(1)},x^{(2)})}{\partial x^{(1)}} = \frac{\partial t^{\rm ovr}(x^{(1)},x^{(3)})}{\partial x^{(1)}}\,; \tag{2}$$

the superscripts "eff" and "ovr" refer to the traveltimes of the dipping event and the overburden reflection, respectively. The identical ray parameters mean that the reflections $x^{(1)}TQRx^{(2)}$ and $x^{(1)}Tx^{(3)}$ share the same downgoing leg $x^{(1)}T$ in the overburden (Figure 1).

Repeating this procedure at point $x^{(2)}$ and matching the time slopes,

$$\frac{\partial t^{\rm eff}(x^{(2)},x^{(1)})}{\partial x^{(2)}} = \frac{\partial t^{\rm ovr}(x^{(2)},x^{(4)})}{\partial x^{(2)}}\,, \tag{3}$$

we find the overburden reflection $x^{(2)}Rx^{(4)}$ that has the same leg $Rx^{(2)}$ as the dipping event $x^{(1)}TQRx^{(2)}$.

Since the overburden is laterally homogeneous and has a horizontal symmetry plane, the raypath of any pure-mode wave reflected from the bottom of the overburden is symmetric with respect to the reflection point. Therefore, the interval traveltime in the layer above the dipping reflector can be expressed as

$$t^{\rm int}(T,R) = t^{\rm eff}(x^{(1)},x^{(2)}) - \frac{1}{2}\left[t^{\rm ovr}(x^{(2)},x^{(4)}) + t^{\rm ovr}(x^{(1)},x^{(3)})\right]. \tag{4}$$

The symmetry of the reflection raypath in the overburden also implies that the interval traveltime is obtained for the "source-receiver" pair $[T,R]$ with the horizontal coordinates

$$T = \frac{\left(x^{(1)}+x^{(3)}\right)}{2}\,,\quad R = \frac{\left(x^{(2)}+x^{(4)}\right)}{2}\,. \tag{5}$$

Essentially, the algorithm outlined above performs kinematic downward continuation of the wavefield through a laterally homogeneous overburden. As in the PP+PS=SS method, this continuation procedure does not require knowledge of the velocity model and is not

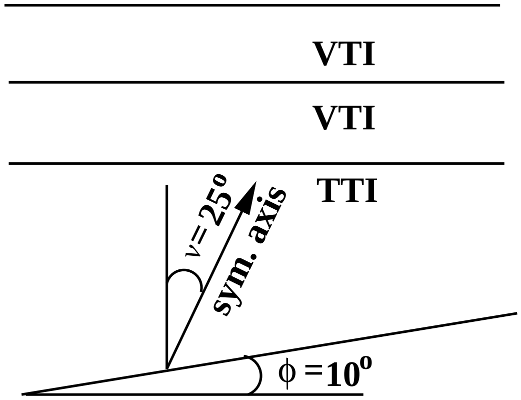

Figure 2. Transversely isotropic model used to test the layer-stripping algorithm for pure modes. The first layer has a vertical symmetry axis (VTI medium) and the following parameters: the symmetry-direction P-wave velocity $V_{P0} = 2$ km/s, the symmetry-direction S-wave velocity $V_{S0} = 1$ km/s, the thickness $z = 0.25$ km, and Thomsen anisotropy parameters $\epsilon = 0.2$ and $\delta = 0.1$. The second layer is also VTI with $V_{P0} = 4$ km/s, $V_{S0} = 2$ km/s, $z = 0.25$ km, $\epsilon = 0.15$, and $\delta = 0.05$; the third layer is dipping TTI with the symmetry axis tilted at $\nu = 25°$, the dip of the bottom $\phi = 10°$, $V_{P0} = 4$ km/s, $V_{S0} = 2$ km/s, $z = 0.5$ km, $\epsilon = 0.25$, and $\delta = -0.05$. The Thomsen parameters in the TTI layer are defined with respect to the symmetry axis (Dewangan and Tsvankin, 2004a,b).

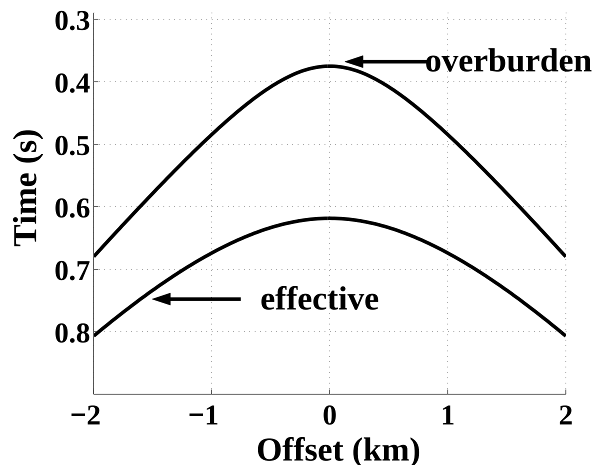

Figure 3. CMP gathers of the PP-wave reflections from the bottom of the second layer (marked "overburden") and the dipping reflector ("effective") for the model in Figure 2. The traveltimes were generated by an anisotropic ray-tracing code.

restricted to isotropic media. Also, in contrast to the Dix-type equation (1), our algorithm can be applied to long-offset data and can handle a laterally heterogeneous target layer.

2.2 Numerical example

The layer-stripping algorithm was tested on PP-wave reflection data from the layered TI model in Figure 2. The traveltimes from the dipping reflector and the bottom of the overburden were computed by anisotropic ray tracing with a shot spacing of 25 m and a receiver spacing of 100 m (Figure 3). To conform with the 2D assumptions of the algorithm, the incidence plane coincides with the dip plane of the reflector and contains the symmetry axis of the TTI layer.

In our implementation of the layer stripping we follow the version of the PP+PS=SS method developed by Grechka and Dewangan (2003) and Dewangan and Tsvankin (2004a,b). For a given pair of points $[x^{(3)}, x^{(4)}]$, this algorithm searches for the coordinates $x^{(1)}$ and $x^{(2)}$ that minimize the interval traveltime in equation (4). This procedure was shown to produce the same results as the methodology based on reflection slopes discussed above (Dewangan and Tsvankin, 2004a,b). Substitution of the estimated coordinates $x^{(1)}$ and $x^{(2)}$ into equations (4) and (5) yields the interval traveltime in the target layer for the source and receiver located at points T and R. Although the depth of the top of the target layer is unknown, the goal of the layer stripping is achieved by obtaining the *horizontal* source/receiver coordinates needed to construct the interval traveltime function.

By repeating the above procedure to cover the whole recorded range of the source-receiver offsets for the dipping event, we compute the interval PP-wave traveltime for a number of the corresponding source/receiver pairs. These pairs do not necessarily form a common-midpoint (CMP), common-shot, or common-receiver gather and need to be sorted to analyze the interval traveltime in any desired configuration. This represents a complication compared with the PP+PS=SS method, in which the type of the output gather can be specified in advance.

To verify the accuracy of the layer stripping, we computed the interval PP-wave traveltime in the dipping layer and the corresponding source/receiver coordinates using ray tracing. The agreement between our method and the ray-tracing results is excellent (Figures 4 and 5). This and other synthetic tests we performed for a representative set of layered TI models confirm that the layer-stripping algorithm is exact and can be applied for large source-receiver offsets.

3 LAYER STRIPPING FOR MODE-CONVERTED WAVES

3.1 Layer-stripping algorithm

In contrast to the generalized Dix equations, our layer-stripping algorithm can be easily adapted for mode-converted (PS or SP) waves. Using the same model assumptions as those in the previous section, we consider

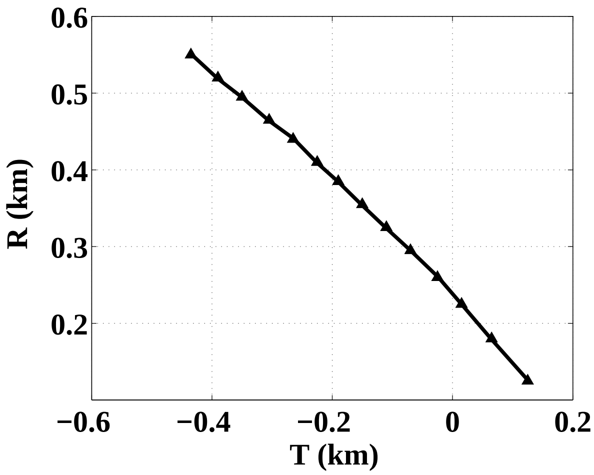

Figure 4. Source and receiver coordinates at the top of the target layer obtained for the model in Figure 2 for a range of surface points $[x^{(3)}, x^{(4)}]$. Here and in Figure 5 the triangles are the output of the layer-stripping algorithm; the solid line marks the results of ray tracing.

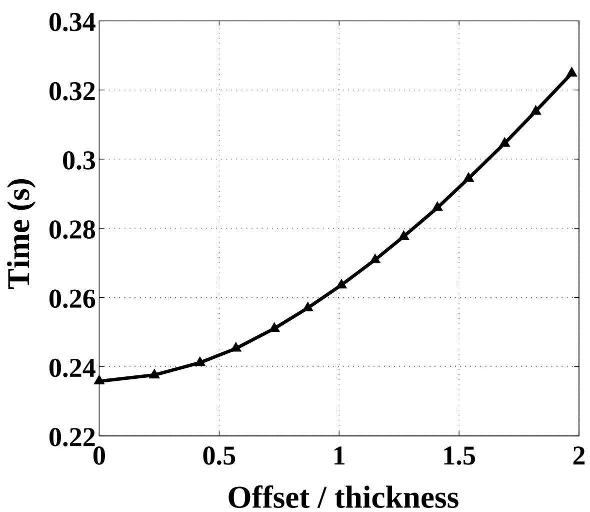

Figure 5. Interval PP-wave traveltime in the target layer as a function of offset for the source/receiver pairs from Figure 4.

the PS-wave converted at a dipping reflector overlaid by a stack of horizontal layers with a horizontal symmetry plane (Figure 6). Since the upgoing leg of the PS mode represents a shear wave, the algorithm has to operate with both PP- and SS-waves reflected from the bottom of the overburden. In the absence of shear-wave excitation, the needed SS traveltimes ($t^{\rm ovr}_{SS}$) can be obtained by applying the PP+PS=SS method to the PP and PS data reflection data (Grechka and Tsvankin, 2002b; Grechka and Dewangan, 2003).

As in the previous section, we consider the reflection raypath $x^{(1)}TQRx^{(2)}$ from the dipping interface,

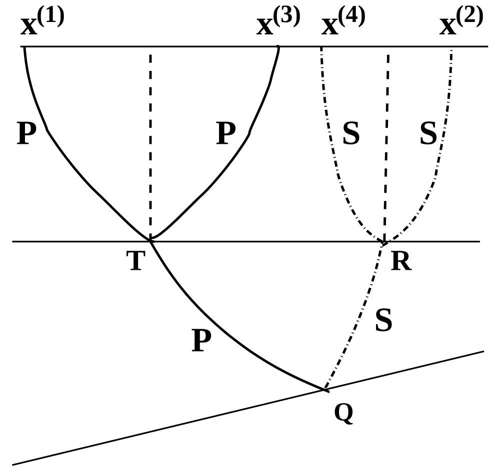

Figure 6. 2D ray diagram of the layer-stripping algorithm for PS-waves. The model is the same as that in Figure 1, with the overburden (the section above points T and R) composed of laterally homogeneous layers with a horizontal symmetry plane. The PS reflection from the dipping interface ($x^{(1)}TQRx^{(2)}$) and the PP reflection from the bottom of the overburden ($x^{(1)}T\,x^{(3)}$) share the same downgoing leg ($x^{(1)}T$). The upgoing leg of the dipping PS event ($R\,x^{(2)}$) coincides with a leg of the overburden SS reflection $x^{(2)}R\,x^{(4)}$.

but now the downgoing leg represents a P-wave, while the upgoing leg is an S-wave (Figure 6). By matching the time slopes at point $x^{(1)}$, we identify the overburden PP-wave reflection $x^{(1)}T\,x^{(3)}$ that shares the segment $x^{(1)}T$ with the dipping PS event. The same procedure at point $x^{(2)}$ yields the reflected SS-wave $x^{(4)}R\,x^{(2)}$ that has the same shear-wave segment $R\,x^{(2)}$ as the PS-wave. Then the source/receiver coordinates T and R for the PS-wave propagating in the target layer can be found from equation (5), while the interval PS-wave traveltime can be expressed as

$$t^{\rm int}_{PS}(T,R) = t^{\rm eff}_{PS}(x^{(1)},x^{(2)}) - \frac{1}{2}\left[t^{\rm ovr}_{PP}(x^{(1)},x^{(3)}) + t^{\rm ovr}_{SS}(x^{(2)},x^{(4)})\right] . \qquad (6)$$

Our implementation of this layer-stripping algorithm for PS-waves is similar to that described above for PP-waves.

Figure 7 shows ray-traced CMP gathers of the target PS event and the pure-mode reflections from the bottom of the overburden for the model in Figure 2. Note the the PS-wave moveout is asymmetric (i.e., the traveltime does not stay the same when the source and receiver positions are interchanged) because of combined influence of the reflector dip ($\phi = 10^\circ$) and the tilt of the symmetry axis ($\nu = 25^\circ$). This moveout asymmetry, however, is handled by our layer-stripping method that relies only on the symmetry of the reflection raypaths of the pure-mode reflections in the overburden.

The layer-stripped interval PS traveltimes and the corresponding source/receiver coordinates are close to

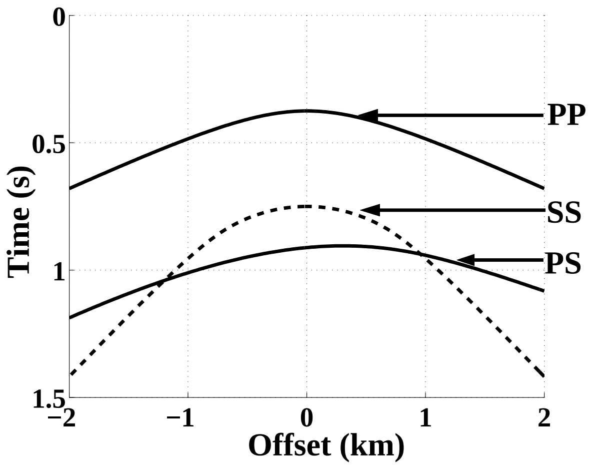

Figure 7. CMP gathers of reflected waves for the model in Figure 2 computed by anisotropic ray tracing. The PS-wave is converted at the dipping interface, while the PP- and SS-waves are reflected from the bottom of the overburden.

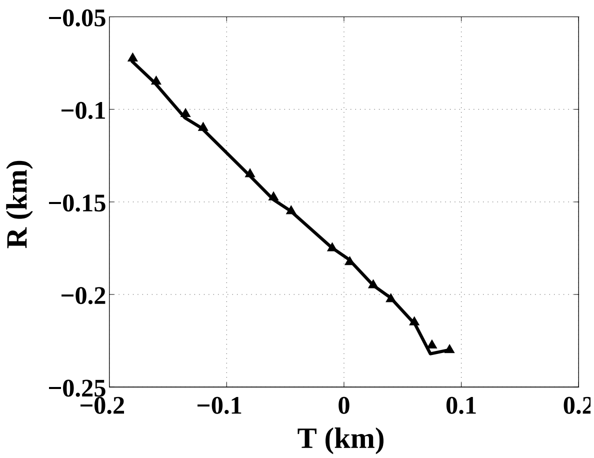

Figure 8. Source and receiver coordinates of the PS-wave at the top of the target layer obtained for the model in Figure 2. Here and in Figure 9 the triangles are the output of the layer-stripping algorithm applied to the data in Figure 7; the solid line marks the results of ray tracing.

the exact values computed by ray tracing (Figures 8 and 9). The minor deviations from the ray-tracing results are caused by interpolation errors related to the finite source and receiver sampling. The PS-wave traveltime function for the target layer, supplemented by the interval PP- and PS-wave moveouts, can serve as the input to the inversion algorithm of Dewangan and Tsvankin (2004b) designed to estimate the parameters of dipping TTI layers.

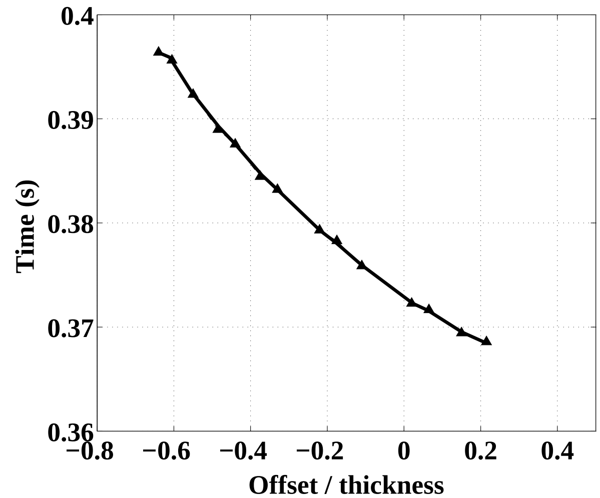

Figure 9. Interval PS-wave traveltime in the target layer as a function of offset for the source/receiver pairs from Figure 8.

4 DISCUSSION AND CONCLUSIONS

The principle of the PP+PS=SS method of Grechka and Tsvankin (2002b) and Grechka and Dewangan (2003) can be used to carry out exact layer stripping for dipping events in anisotropic media. The main assumptions of the algorithm introduced here are that the overburden is laterally homogeneous and has a horizontal symmetry plane (i.e., up-down symmetry) in each layer. The target layer above the dipping reflector, however, is allowed to be laterally heterogeneous without up-down symmetry, although the incidence plane has to coincide with a vertical symmetry plane for the whole model.

Under these assumptions, simple operations with reflection traveltimes can be used to identify the overburden events that have the same up- and downgoing legs as the reflection from the dipping interface. This allows us to perform kinematic downward continuation of the wavefield and obtain the interval traveltime-offset function without knowledge of the medium parameters. Numerical examples for layered transversely isotropic media with a vertical and tilted symmetry axis confirm that the algorithm gives exact results for both pure and converted modes. Although the testing was limited to PP- and PS-waves, the methodology can be also used for layer-stripping of SS and SP reflections excited by shear sources.

In contrast to the existing layer-stripping techniques that employ the generalized Dix equation, our algorithm is not restricted to the hyperbolic portion of the moveout curve. Therefore, it can yield exact *long-offset* interval traveltimes of both horizontal and dipping events in symmetry planes of anisotropic media. Note that existing layer-stripping methods for long-spread data are derived for the quartic moveout coefficient in layer-cake models and cannot handle reflector dip (Tsvankin, 2001).

The 2D algorithm discussed here can be extended to wide-azimuth data using the 3D version of the PP+PS=SS method outlined by Grechka and Tsvankin (2002b) and Grechka and Dewangan (2003). It should be mentioned, however, that our methodology operates with individual traveltimes, which makes it more complicated and computer-intensive than the Dix-type layer stripping. Hence, if the velocity model of the overburden is known and only conventional-spread P-wave data are available, it is more efficient to apply the generalized Dix equations of Alkhalifah and Tsvankin (1995) and Grechka et al. (1999).

An important application of the above results is in velocity analysis for tilted TI layers using multicomponent (PP and PS) data. Dewangan and Tsvankin (2004a,b) showed that the asymmetry attributes of PS-waves, combined with pure-mode moveout signatures, can provide sufficient information for parameter estimation in a homogeneous TTI medium. The layer-stripping algorithm introduced here can help to implement their inversion technique for realistic vertically heterogeneous models with a stratified overburden above the dipping target TTI layer.

Our method can also help to overcome the limitations of the generalized Dix equation in the dip-moveout inversion for the time-processing parameter η in VTI media. Because of the need to compute the interval NMO velocities in the overburden for non-existent reflectors, Dix-type algorithms designed to estimate η using dipping events have to rely on the presence of both horizontal and dipping interfaces in each layer (Alkhalifah and Tsvankin, 1995; Tsvankin, 2001). This requirement, which is often difficult to satisfy in practice, can be removed by replacing the generalized Dix equation with our velocity-independent layer-stripping technique.

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