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Modeling and imaging with isochron rays

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ABSTRACT

Isochron rays are lines perpendicular to isochrons, which represent surfaces of constant two-way traveltime. The image of a temporal sequence of seismic impulses is a sequence of isochrons in depth. The later the time impulse the deeper the isochron. The term isochron ray arises from an analogy between the isochron "movement" and the wave propagation. While isochrons behave as wavefronts, its perpendicular lines can be regarded as rays. The speed of the isochron movement depends on the medium velocity and the source-receiver position. We introduce the term equivalent-velocity to refer to the speed of isochron movement. In the particular case of zero-offset data, the equivalent velocity is half of the medium velocity. We use the concepts of isochron-rays and equivalent velocity to extend the application of the exploding reflector model to non-zero offset imaging problems. In particular, we employ these concepts to extend the use of zero-offset wave-equation algorithms for modeling and imaging common-offset sections. In our imaging approach, the common-offset migration is implemented as a trace-by-trace algorithm in three steps: 1) equivalent velocity computation, 2) data-conditioning for zero-offset migration, and 3) zero-offset wave-equation migration. We apply this methodology for modeling and imaging synthetic common-offset sections using two kinds of algorithms: finite-difference and split-step wavefield extrapolation. We illustrate the isochron-ray imaging methodology with a field-data example and compare the results with conventional common-offset Kirchhoff migration. This methodology is attractive because (1) it permits depth migration of common-offset sections or just pieces of that by using wave-equation algorithms, (2) it extends the use of robust zero-offset algorithms, (3) it presents favorable features for parallel processing, (4) it permits the creation of hybrid migration algorithms, and (5) it is appropriate for migration velocity analysis.

Key words: wave equation, imaging, modeling, isochron, isochron-rays

1 INTRODUCTION

Isochron rays are curves associated with propagating isochrons, that is, with surfaces that are related to seismic reflections with the same two-way traveltime. Isochron surfaces play an important role in seismic imaging because they are closely related to the impulse response of depth migration. Hubral *et al.* (1996) showed how a weighted Kirchhoff-type isochron-stack integral can be applied to true-amplitude algorithms for both modeling and data transformation. The general theory of data mapping, presented by Bleistein *et al.* (2000), emphasizes the importance of isochrons in the establishment of integral formulas for inversion.

Iversen (2004) introduced the term isochron ray for trajectories associated with surfaces of equal two-way time, i.e. isochron surfaces, and suggested the potential use of isochron rays in future implementations of prestack depth migration. Here we exploit the idea and present a methodology that makes use of the isochron ray concept to perform prestack depth migration. We consider as isochron rays the curves that are perpendicular to the isochrons associated with the image produced by the migration of a single finite-offset seismic trace. That is, isochron rays are the orthogonal trajectories to isochrons. This concept differs from the one introduced by Iversen (2004), which involves non-orthogonal trajectories.

Our imaging approach consists of a trace-by-trace algorithm, wherein each finite-offset input trace is first conditioned for zero-offset extrapolation and then migrated using an equivalent-velocity model. We present two imaging strategies. In the first approach the prestack depth migration is achieved by performing the downward continuation of the conditioned data along the isochron rays, followed by the application of the zero-offset imaging condition. The second imaging approach consists of a reverse-time migration algorithm, where the conditioned data is reversed and injected into the equivalent velocity model along an isochron defined by the time-shift previously applied on the input trace.

The main purpose of this research is the development of a methodology to apply zero-offset wave-equation algorithms to solve finite offset problems. In particular, this methodology can be useful in the implementation of migration velocity analysis methods based on offset continuation, see Silva (2005). Also, the presented methodology is attractive because (1) it permits depth migration of common-offset gathers using wave-equation algorithms, (2) it extends the use of robust zero-offset algorithms to the common-offset case, (3) it is based on algorithms that are appropriate for parallel processing, and (4) it permits to combine different imaging algorithms.

2 THE ISOCHRON RAY CONCEPT

Given a source-receiver pair and a fixed reflection traveltime, the related isochron is the surface that answers the question: Where are the possible reflection points located? In other words, an isochron surface is the image of points with the same reflection time. An isochron surface can be viewed as a hypothetical reflector whose reflections from the source S are recorded simultaneously at the receiver R. For any point M belonging to an isochron surface, the traveltime measured along the path SMR does not vary.

The isochron surfaces play an important role in seismic imaging, especially in prestack depth migration. For a single trace composed of a sequence of impulses, the image produced by depth migration is represented by a set of isochrons. The longer the reflection time, the greater the distance between the source-receiver pair and the isochron. The shape of the isochrons depends on the velocity field, the reflection time, and the spatial location of the source and receiver. In a homogeneous medium, every isochron has an ellipsoidal shape whose focus points are located at the source and receiver position, while the eccentricity is defined by the seismic wave velocity and the reflection time. The shallowest isochron surface tends to collapse into a straight line connecting the source-receiver pair.

Consider a sequence of depth migrated images where the input data consist of a sequence of seismic impulses with varying reflection time. Notice in Figure 1(a) as the impulse time increases by the same amount with the first being just little longer than the direct arrival traveltime. The initial surface (first isochron) observed in the first image moves in depth, changing its shape and acting as a propagating wavefront. If a point of the initial surface is selected and followed during the sequence, its trajectory will define a curve. We refer to this curve as an isochron ray because it acts as a ray, while the moving isochron plays the role of a propagating wavefront.

2. 1 Equivalent velocity media

The propagating isochron "moves" through the model with a speed that is different from the wave propagation velocity. The isochron propagation velocity depends on the source-receiver location, and the medium velocity and it varies even in isotropic-homogeneous media.

For a given source-receiver pair, we can imagine a hypothetical medium with the velocity distribution identical to that of the isochron velocity propagation. We refer to this as the equivalent velocity medium. There are two features that characterize the equivalent velocity field. First, isochron propagation velocity depends on the isochron ray direction, which means it can be multivalued in the presence of caustics. Second, equivalent velocity fields present a singularity in the line connecting the source-receiver pair, which corresponds to the hypothetical starting isochron.

Depending on the complexity of the original velocity model, we distinguish two cases of determining equivalent velocity and isochron ray tracing. One is based on the assumption of the absence of caustics, and another is the general case, where no restrictions are imposed on the velocity model.

Let us assume a smooth seismic velocity model with no caustics. Consider a source-receiver pair located in a horizontal plane where the velocity does not vary in a small slab between the source and the receiver. In this situation, the isochron-field can be reproduced by a hypothetical experiment where the seismic source is a horizontal segment connecting the source-receiver pair, and the seismic velocity is the equivalent velocity medium. All the isochron rays are perpendicular to the line source. In the vertical plane that contains the source-receiver pair, all the isochron rays are vertical at their starting point. The isochron rays obey Snell's law and the wavefronts are the surfaces of constant traveltime in space satisfying the eikonal equation:

$$\left(\frac{\partial t_{sr}}{\partial x}\right)^2 + \left(\frac{\partial t_{sr}}{\partial y}\right)^2 + \left(\frac{\partial t_{sr}}{\partial z}\right)^2 = \frac{1}{V_{eq}^2(x,y,z)}. \quad (1)$$

In the absence of triplications, the equivalent velocity medium can be directly determined by applying the eikonal equation on the two-way traveltime map. This velocity field can be used to migrate the conditioned data using conventional zero-offset migration algorithms. In the presence of triplications, equivalent velocity media are multivalued and cannot be derived from traveltime maps. In this case, the isochron propagation velocity should be represented in isochron ray coordinates and has to be computed using a proper isochron ray-tracing algorithm. Also, the migration should use wave-field extrapolation in isochron ray coordinates. This case falls outside the scope of this paper and remains subject to future reseach.

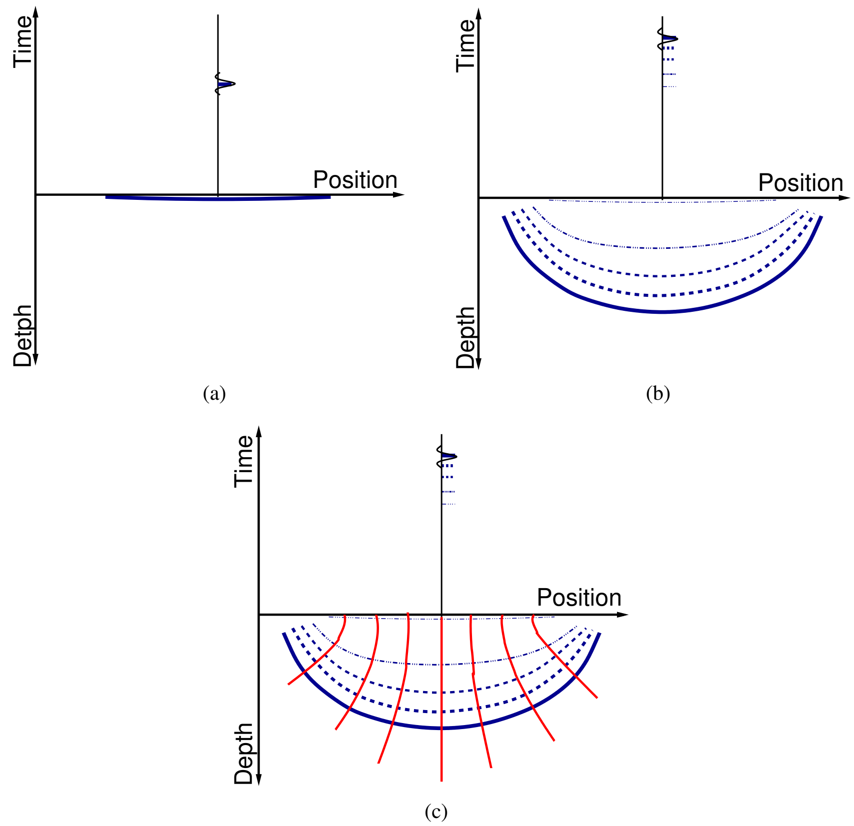

Figure 1. Propagating isochrons

2. 2 Isochron ray-tracing without media restriction

The isochron propagation can be performed by applying basic principles of wave propagation. For simplicity, consider an isochron in a 2D media. In Figure 2(a), the point M is the intersection of three curves:

- $z = \zeta_s(x, S, t_s)$ is the wavefront that comes from the source position, $S = (x_s, z_s)$, at the time t_s.
- $z = \zeta_r(x, R, t_r)$ is the wavefront that comes from the receiver position, $R = (x_r, z_r)$, at the time t_r.
- $z = \zeta(x, S, R, t_s + t_r)$ is the isochron corresponding to the source-receiver pair SR at the two-way traveltime $t_{sr} = t_s + t_r$.

Figure 2(b) shows the isochrons after a propagation time of $\delta t/2$, the wavefront $z = \zeta_s(x, S, t_s)$ moves to $z = \zeta_s(x, S, t_s + \delta t/2)$, the wavefront $z = \zeta_r(x, R, t_r)$ moves to $z = \zeta_r(x, R, t_r + \delta t/2)$, the isochron $z = \zeta(x, S, R, t_s + t_r)$ moves to $z = \zeta(x, S, R, t_s + t_r + \delta t)$, and the intersection point moves to M'. In 2D models, the triple intersection point can be found by merely locating the intersection between the source and receiver wavefronts. In this case, an isochron ray can be traced just by mapping the intersection points step by step.

In the case of 3D, the intersection between the source and receiver wavefronts is a curve instead of a point. Consequently the use of the unknown isochron is needed for determining the intersection point. The isochron is unknown but it can be defined as the envelope of intersection lines between source and receiver wavefronts whose total traveltime is constant.

2. 3 The exploding reflector model

The exploding reflector model (Lowenthal *et al.*, 1985) is widely applied in both seismic modeling and imaging algorithms. Although it is an approximation that cannot be reproduced by a physical experiment, it leads to simple, robust and efficient algorithms. Zero-offset seismic data can be modeled and migrated by a large number of approaches, such as Kirchhoff, finite-differences, and Gaussian beams.

For zero-offset data, the two-way traveltime of primary reflections with normal-incidence angle can be computed by tracing normal rays in a half-velocity medium.

The isochron-rays play a role analogous to normal rays, i.e., they are perpendicular to the reflectors, and the traveltime measured along them is the time on the two-way path: source-reflector-receiver. While the normal rays can be traced using the half-velocity medium, the isochron-rays need an equivalent velocity medium that depends on the source and receiver location. Therefore we need to define an equivalent velocity medium for each source-receiver pair. Another important dif-

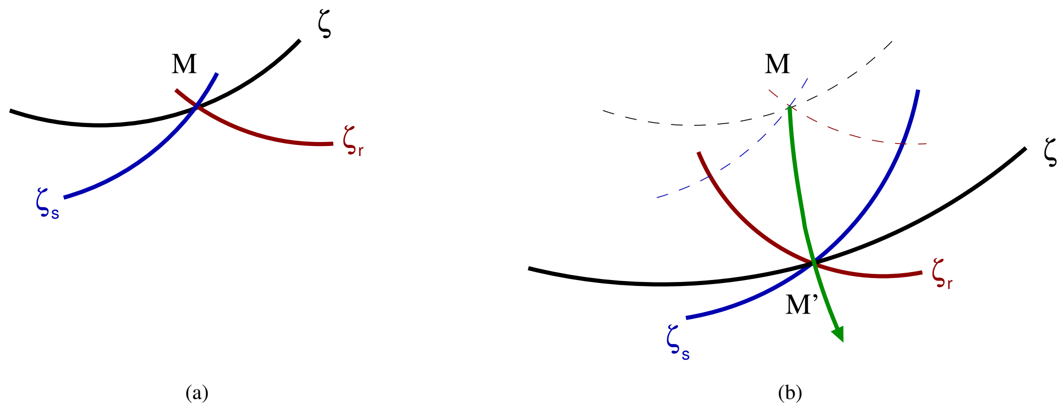

Figure 2. Isochron ray-tracing scheme: a) wavefronts at t_s and t_r, isochron at $t_r + t_s$, b) wavefronts at $t_s + \delta t/2$ and $t_r + \delta t/2$, isochron at $t_r + t_s + \delta t/2$.

ference between normal and isochron rays is the take-off (or emergence) angle. While normal rays can assume any direction at the recording surface even when the first layer is homogeneous, the isochron rays are always perpendicular to it, see figure 3(b). Because of the analogy described above, we suggest the expression "Equivalent exploding reflector model" to refer to the association of isochron rays and equivalent velocity model.

3 IMAGING USING ISOCHRON RAYS

3. 1 Modeling

In this section, we present two examples in which we extend the use of zero-offset algorithms to finite offset data by applying the concepts of isochron rays, equivalent velocity, and exploding reflector model. We generate three common-offset sections with the methods: finite-difference, split-step wavefield extrapolation and Kirchhoff. The third dataset was generated by conventional Kirchhoff modeling to be used as benchmark.

The 2D seismic model consists of six interfaces immersed in a smooth velocity field, where the wave velocity propagation varies from 1500 m/s at the shallow part, to 3000 m/s at the bottom. The Figure 4 shows the velocity model and the interfaces. For all cases, the source-receiver pair is located on the horizontal plane at z=0.

For Kirchhoff modeling, a Ricker wavelet with dominant frequency of 20 Hz was used. The integration was performed using a spatial interval of 1 m without any special care about the dynamic aspects. Figure 5(a) shows the synthetic Kirchhoff common-offset section for the half-offset h=500 m.

For both finite-difference and wavefield extrapolation modeling, the common-offset section was constructed trace-by-trace using an equivalent velocity medium for each CMP position. The equivalent velocity media were defined following the steps:

- Build the travel-time map $t_s(x, z)$ from the source location to all points of the model using an eikonal solver algorithm.
- Do the same from the receiver location, getting the travel-time map $t_r(x, z)$.
- Add the two maps to obtain the two-way travel-time map $t_{sr} = t_s + t_r$.
- Find the spatial partial derivatives of the two-way travel-time (t_{sr}).
- Apply the eikonal equation to determine the equivalent velocity for every grid point.

The equivalent velocity media has a singularity between the source-receiver pair, where the velocity goes to infinity. Thus we need to adopt special procedures to avoid numerical problems in this region. The applied procedures are different for each case. However both are based on an analytical solution for the isochron propagation in the vicinity of the source-receiver pair. In Figure 6 we present the equivalent-velocity field for the central position of the seismic model presented in Figure 4.

For finite difference modeling, we avoid numerical instability by clipping the equivalent velocity field and placing the receivers along an isochron located away from the singularity. A good choice for locating this recording isochron is a region where the wave propagation is constant because it is an ellipse in this case.

The recorded isochron-field contains information from all directions, but only information that travels along isochron rays should be considered. In other words, we have to sum the amplitudes along isochrons, which is equivalent to a stack of the information collected in the recording isochron along the time. Figure 5(b) is the common-offset gather modeled by the finite-difference approach. Each trace of this gather is the result of the stacking of all traces recorded along an isochron whose midpoint between the source and receiver corresponds to the trace location. Figure 7(a) corresponds to the seismogram recorded at the central position of the seismic model.

For modeling by wave-field extrapolation, the adopted procedure consists of recording the wave-field (isochron-field)

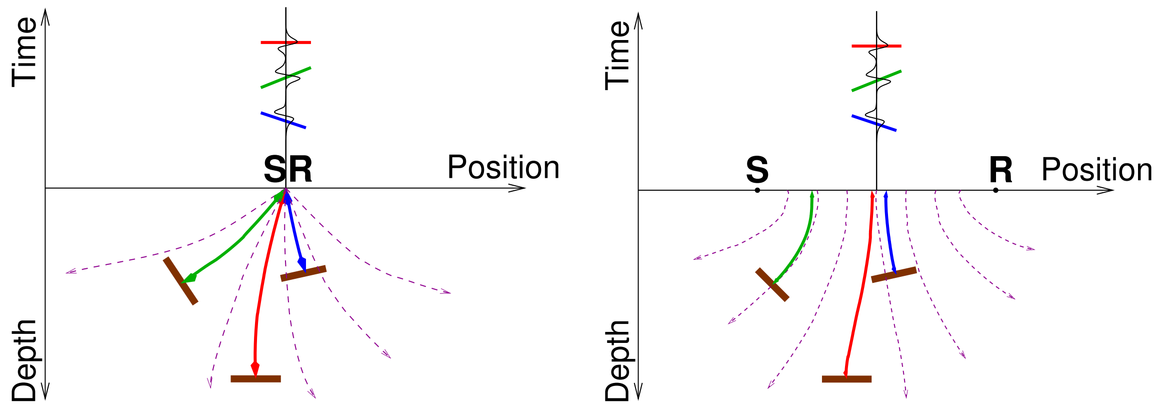

Figure 3. a) zero-offset exploding reflector model, b) common-offset "exploding reflector model"

at a horizontal plane below the singularity. Since the source-receiver pair is located at the surface $z = 0$, the singularity is taken out of the equivalent velocity model just by excluding a tiny slab with a thickness corresponding to the vertical sample interval. The wave-field is extrapolated from the bottom to the one-sample deep surface where it is recorded, see Figure 7(b). After applying a time-shift to compensate for the slab removing, the data are stacked to generate the modeled trace. The initial time of the modeled trace corresponds to the direct arrival traveltime. The procedure described above is independently applied for all desired output positions of the modeled common-offset gather. Alternatively, the problem of avoiding the singularity can be addressed by redatuming the data from the surface to a deeper plane.

3. 2 Migration

In principle, all of the zero-offset migration methods based on the exploding reflector model can have their use extended to finite-offset gathers by making use of isochron rays and equivalent velocity media. In this section, we discuss two cases: migration by wave-field extrapolation (WEM) and reverse time migration (RTM). In both cases, the isochron ray migration can be implemented in a trace-by-trace algorithm. For each trace, the following steps are carried out: 1) computation of the equivalent velocity model, 2) creation of the conditioned data for wave-field reconstruction, 3) migration of the conditioned data by a zero-offset algorithm using the equivalent velocity model, and 4) addition of the migration result to the image.

The conditioning data procedure is not the same for WEM and RTM, but in both cases a half-derivative followed by a time shift is applied to the input trace. For the WEM case, the input trace is time-shifted by a negative amount that corresponds to the traveltime measured along the raypath connecting the source and receiver. The conditioned data gather for WEM is obtained by repeating the shifted trace for each trace position located between the source-receiver, while the remaining positions are filled with zeros. In the RTM case the time-shift is also negative and it corresponds to the time of an isochron where the reverse-data is injected. This isochron should be as far as possible from the singularity. A good choice would be an ellipse when the source-receiver pair is located in a homogeneous region. In addition to the required steps described above, linear spatial tapering is applied to the conditioned data to avoid the presence of artifacts at the final image. another important difference between RTM and WEM using the equivalent exploding reflector model is the imaging condition. In the RTM case, the partial image gather (the result of the migration of a single trace) corresponds to the last snapshot of the wave-field, while this gather is obtained after applying a zero-offset image condition in WEM case.

Figure 8(a) corresponds to the zero-offset image obtained by conventional wave-field extrapolation migration. Figures 8(b) and 8(c) are the common-offset WEM and RTM images, respectively. In both cases, the synthetic Kirchhoff common-offset gather was used as input.

4 APPLICATION TO FIELD-DATA

The WEM using isochron rays was applied in a pseudo 2.5D dataset, which consists of twenty-two common-offset gathers extracted from a 3D dataset via the following sequence. First, the input traces were organized in twenty-two groups, using as sorting criteria the source-receiver offset; second a 3D Kirchhoff time migration algorithm was applied; finally, a 2.5D Kirchhoff time demigration procedure was applied to each image. In the sorting procedure of the first step, each input trace was multiplied by an areal factor in order to compensate for the effect of acquisition irregularities. The weight function used in the 3D time-migration algorithm produces a true-amplitude image gather when the medium velocity is constant, i.e the output amplitudes are proportional to the reflection coefficients. Also, the applied demigration program uses a true-amplitude weight function, that produces a 2D common-offset gather whose amplitudes are affected by a 3D geometrical spreading factor, which is correct when the medium is homogeneous.

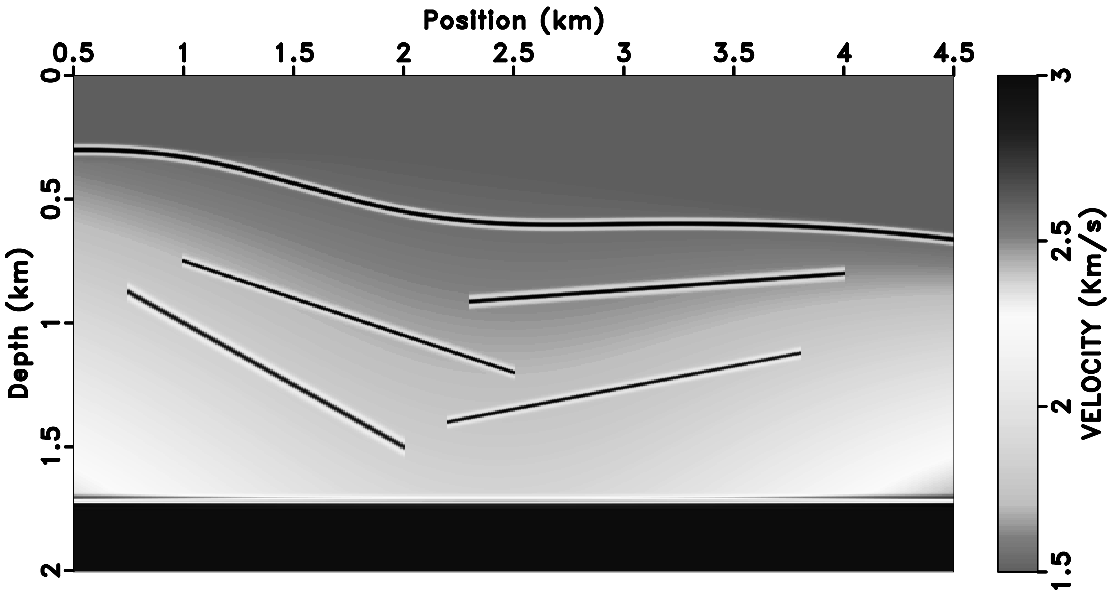

Figure 4. 2D seismic model: superposition of the velocity field and interfaces.

The minimum offset is 160 m and the increment between offsets is 200 m. Each common-offset gather has 1351 traces and the distance between them is 18.75 m. The traces are 5.0 s long and the time sampling interval is 4 ms.

The same smoothed velocity model (figure 11) was used to migrate four common-offset gathers (from 1560 m to 2160 m) using the wave-field extrapolation approach and a traditional common-offset Kirchhoff program. The WEM image from the gather with a larger offset is presented in the figure 9(a), while the figure 9(b) shows the Kirchhoff result. As expected, there are no significant differences between the images from these migration methods. The result is basically the same because we use the same velocity and the same approach to compute traveltime maps for Kirchhoff migration and to calculate the equivalent velocity in wave-field extrapolation A similar result is observed when we stack the four migrated sections: compare Figure 10(a) and Figure 10(b). A significant difference between the Kirchhoff method and the WEM isochron ray migration should be expected in the presence of caustics if we determine the equivalent velocity medium by means of an isochron ray-tracing algorithm.

5 DISCUSSION

The computational cost of modeling or migrating an individual seismic trace using the isochron ray approach as presented above is close to the cost of modeling or migrating a zero-offset section. Besides the high computational cost, the described methodology is restricted to smooth velocity models, which reduces the attractiveness of this approach. The computational cost can be reduced by using beams, redatuming, and limited aperture. Silva & Sava (2008) show that the combination of these procedures can drastically decrease the processing time, especially for greater offsets. Problems due to triplication can be eliminated by representing the equivalent velocity in isochron ray coordinates.

Future research includes the development of an effective isochron ray tracing algorithm without making any assumption about the medium. This algorithm might be based in the algorithm described in the section isochron ray-tracing, which should be implemented by making use of the paraxial ray theory. The isochron ray-tracing algorithm could be used to define equivalent velocity media in isochron ray coordinates, which can be used to extrapolate the isochron-field in this coordinate system (Sava & Fomel, 2005).

6 CONCLUSION

We apply, for the first time, the concept of isochron rays to modeling and imaging of seismic data. We introduced the concept of equivalent velocity to extend the use of the exploding reflector model to non-zero-offset data. We show how to use this concept for modeling and imaging of single finite-offset traces in 2-D media using two kind of algorithms: finite difference and wave-field extrapolation.

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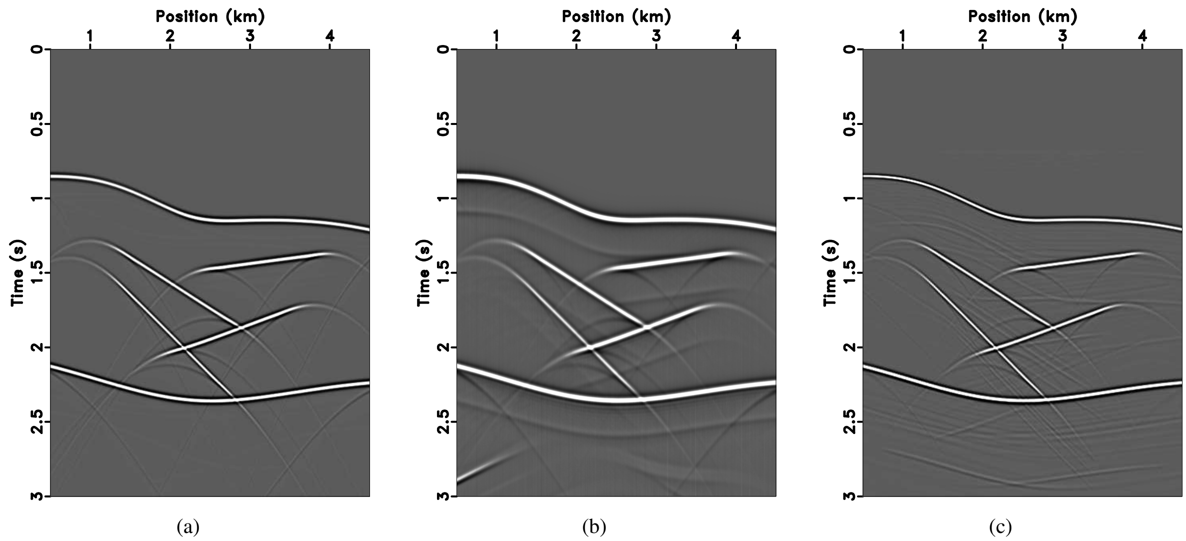


Figure 5. Common-offset gathers: a) Conventional Kirchhoff modeling, b) isochron-ray finite-difference modeling, and c) isochron-ray wavefield extrapolation modeling.

REFERENCES

Bleistein, N., Cohen, J. K., and Stockwell, J., 2000, Mathematics of multidimensional seismic imaging, migration and inversion: , number 13, Interdisciplinary applied mathematics.

Hubral, P., Schleicher, J., and Tygel, M., 1996, A unified approach to 3-D seismic reflection imaging, part I: Basic concepts: Geophysics, **61**, no. 03, 742–758.

Iversen, E., 2004, The isochron ray in seismic modeling and imaging: Geophysics, **69**, no. 4, 1053–1070.

Lowenthal, D., Lu, L., Roberson, R., and Sherwood, J. W. C., 1985, The wave equation applied to migration *in* Gardner, G. H. F., Ed., Migration of seismic data:: Soc. of Expl. Geophys., 208–227.

Sava, P., and Fomel, S., 2005, Riemannian wavefield extrapolation: Geophysics, **70**, no. 03, T45–T56.

Silva, E. F. F., and Sava, P., 2008, Accelerating wavefield extrapolation isochron-ray migration:.

Silva, E. F. F., 2005, Horizon velocity analysis using oco rays: Horizon velocity analysis using oco rays:, SBGf, 9th Congress of The Brazilian Society of Geophysics, Salvador, Brazil.

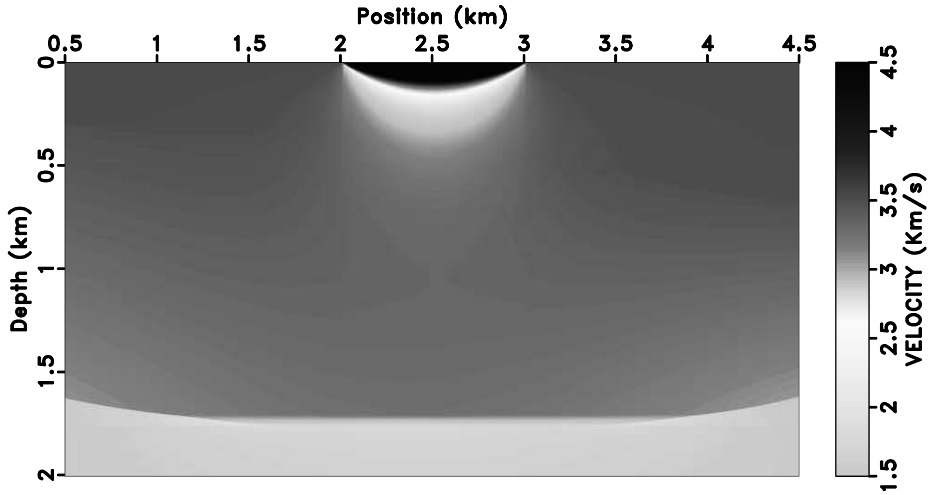
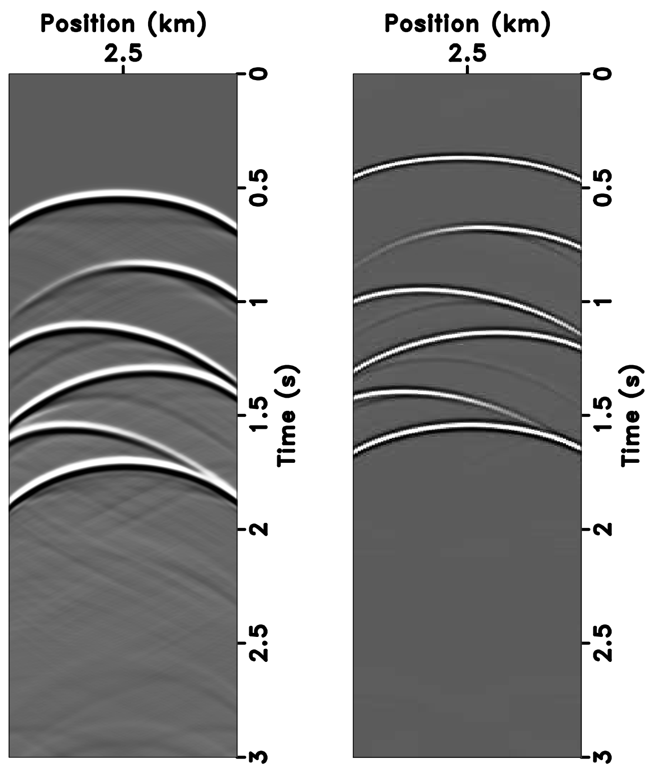
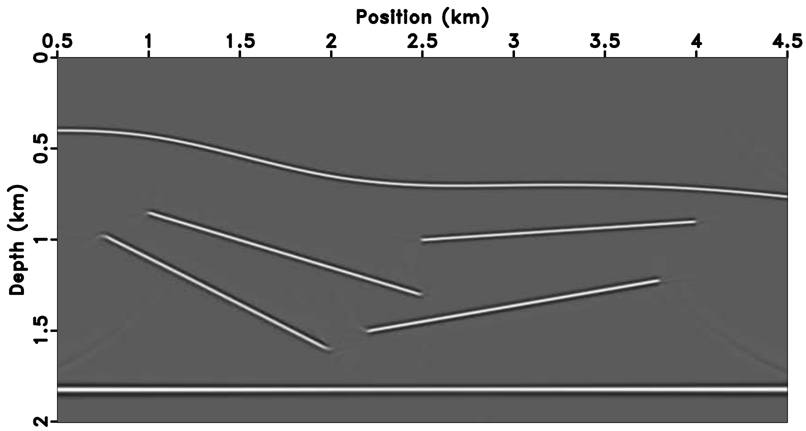


Figure 6. Equivalent velocity for the central position of the seismic model clipped at 4,5 m/s.

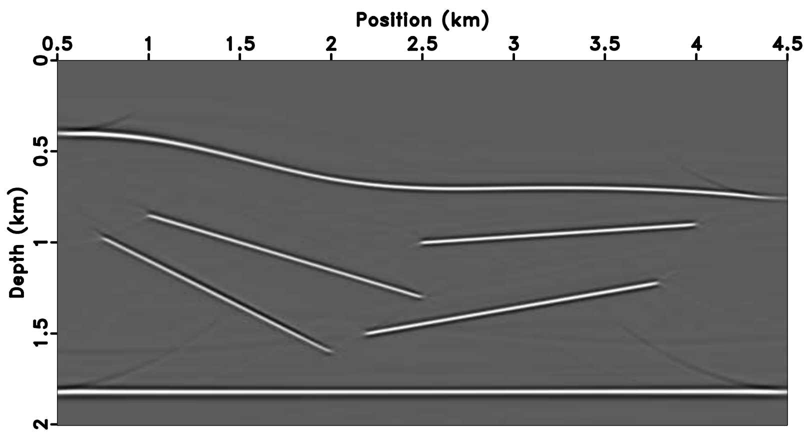


(a) (b)

Figure 7. a) Finite-difference recorded isochron field, b) Extrapolated isochron field recorded at $z = 0.005km$



(a)



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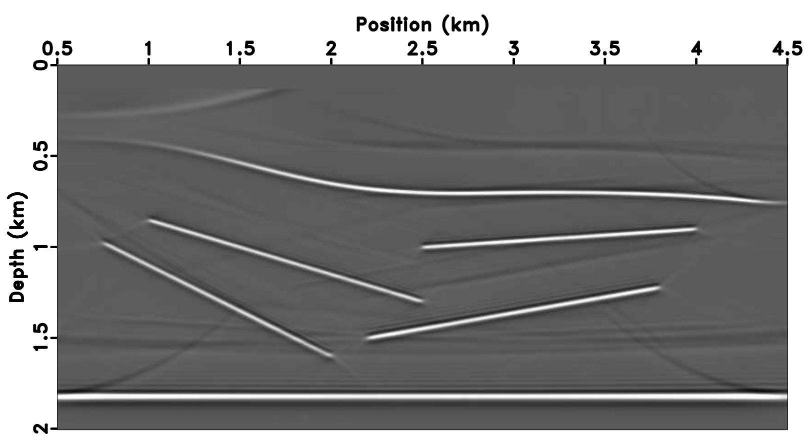
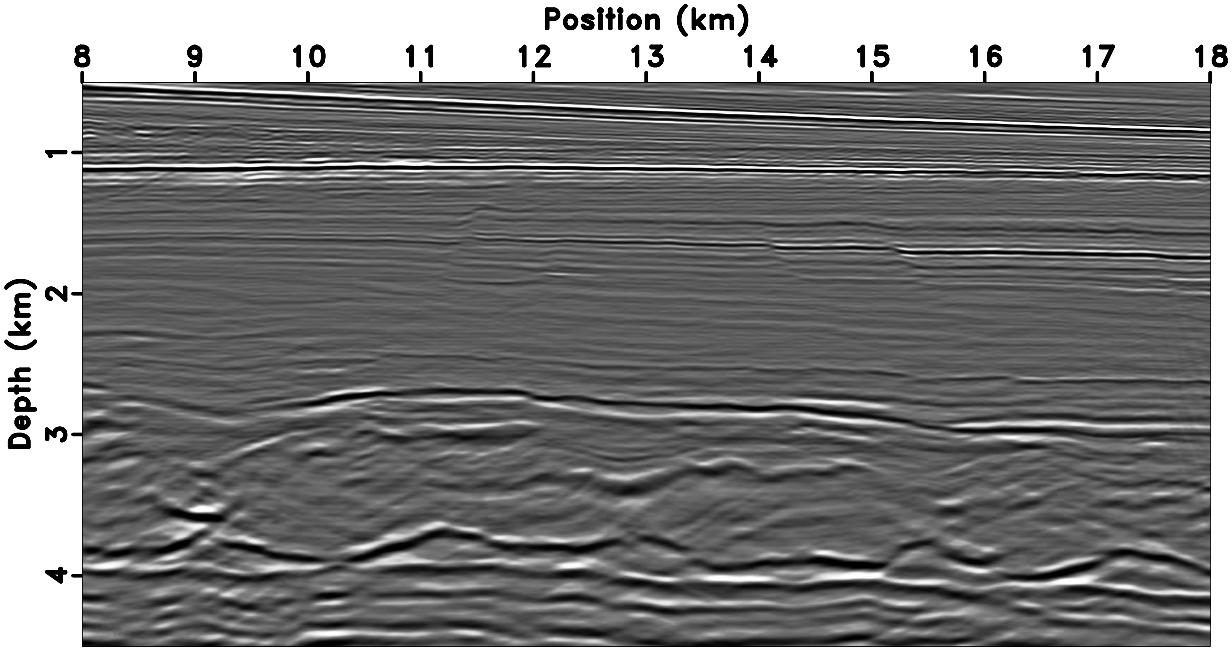


(c)

Figure 8. a) Zero-offset wave-field extrapolation migration, b) Common-offset wave-field extrapolation migration, c) Common-offset reverse time migration,



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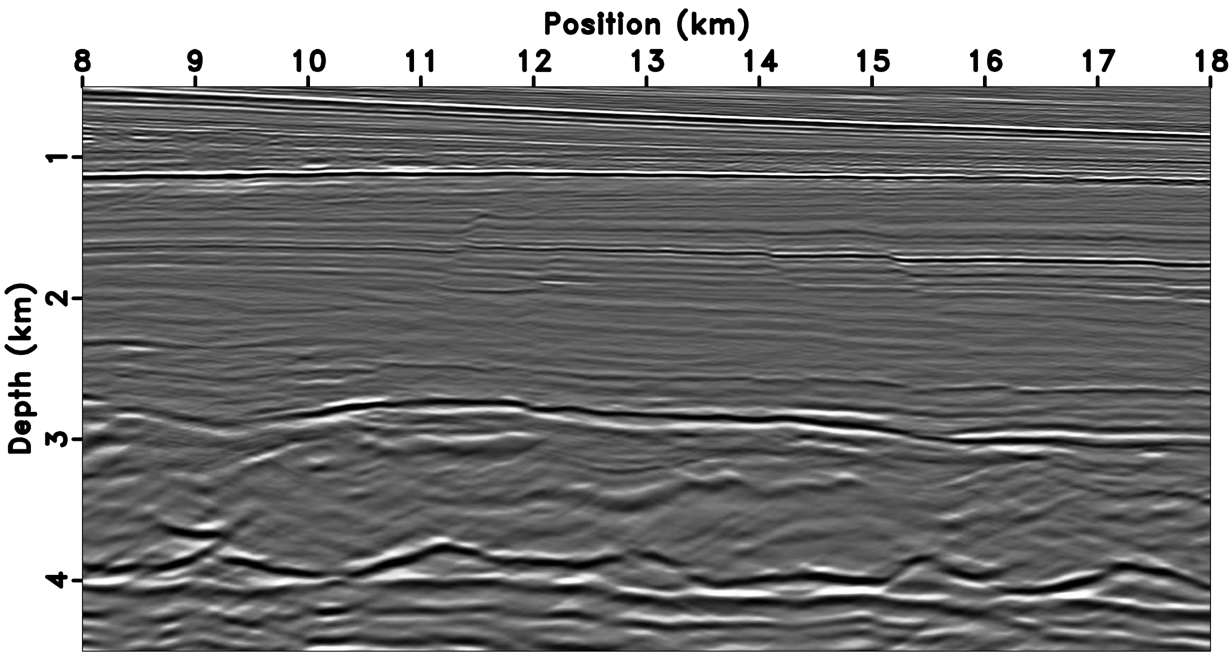
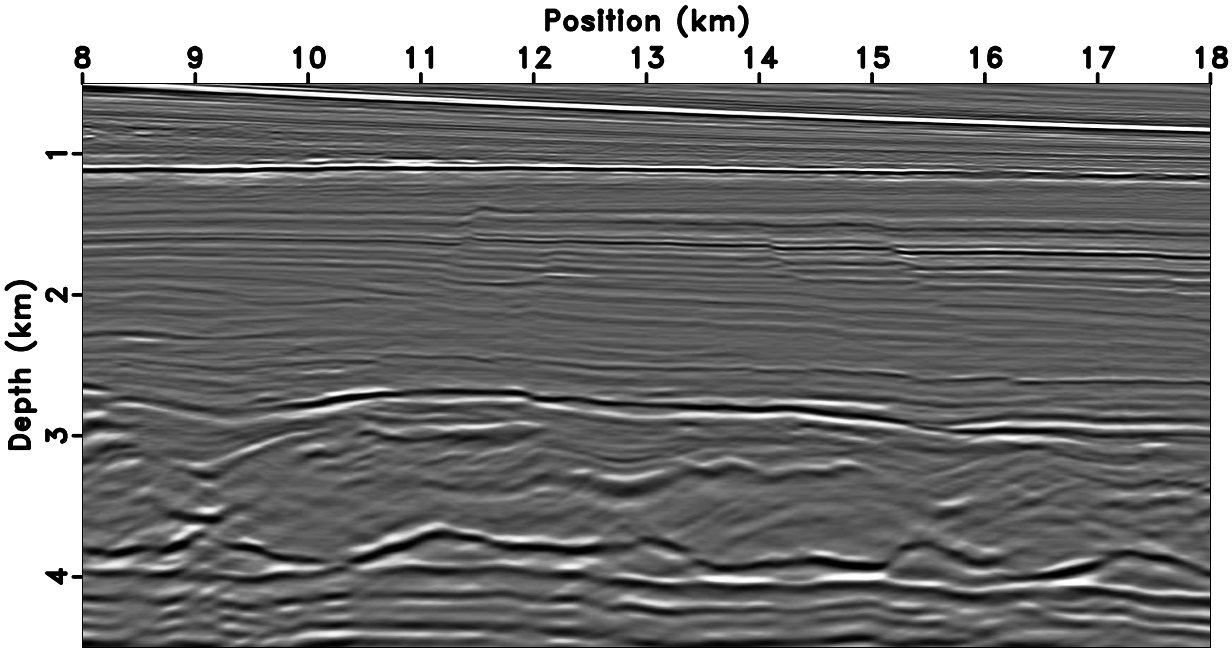


(b)

Figure 9. Common-offset images: a) wavefield isochron-ray migration, b) Kirchhoff migration



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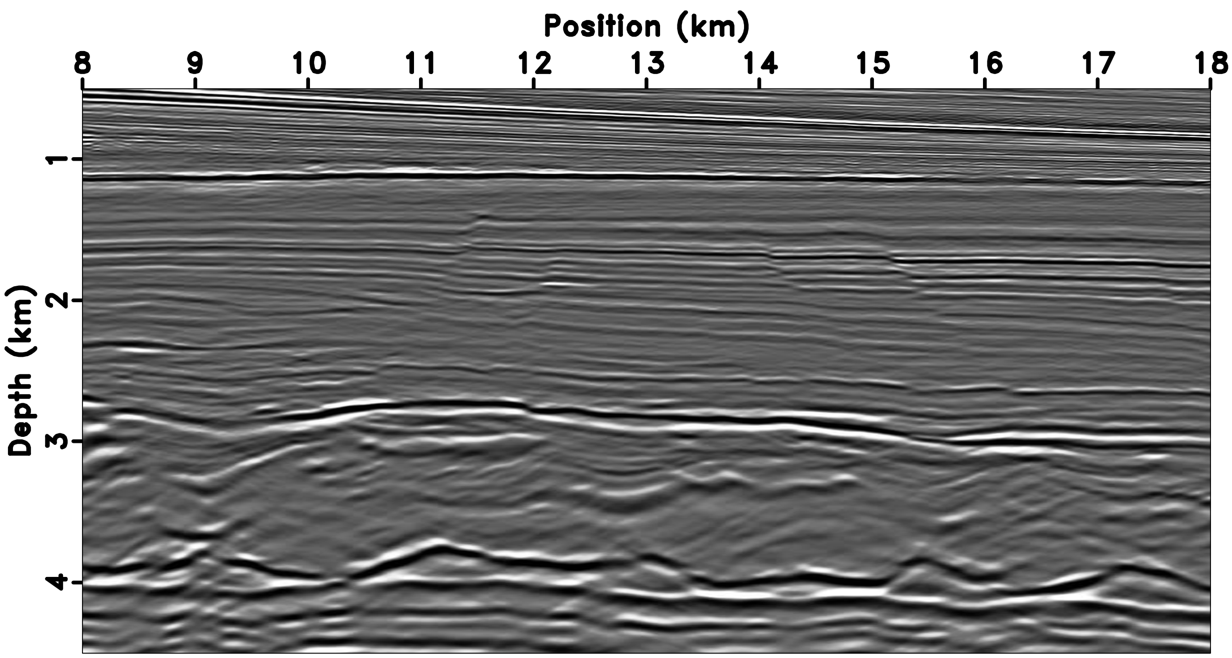

(b)

Figure 10. Stack of common-offset images: a) wavefield isochron-ray migration, b) Kirchhoff migration

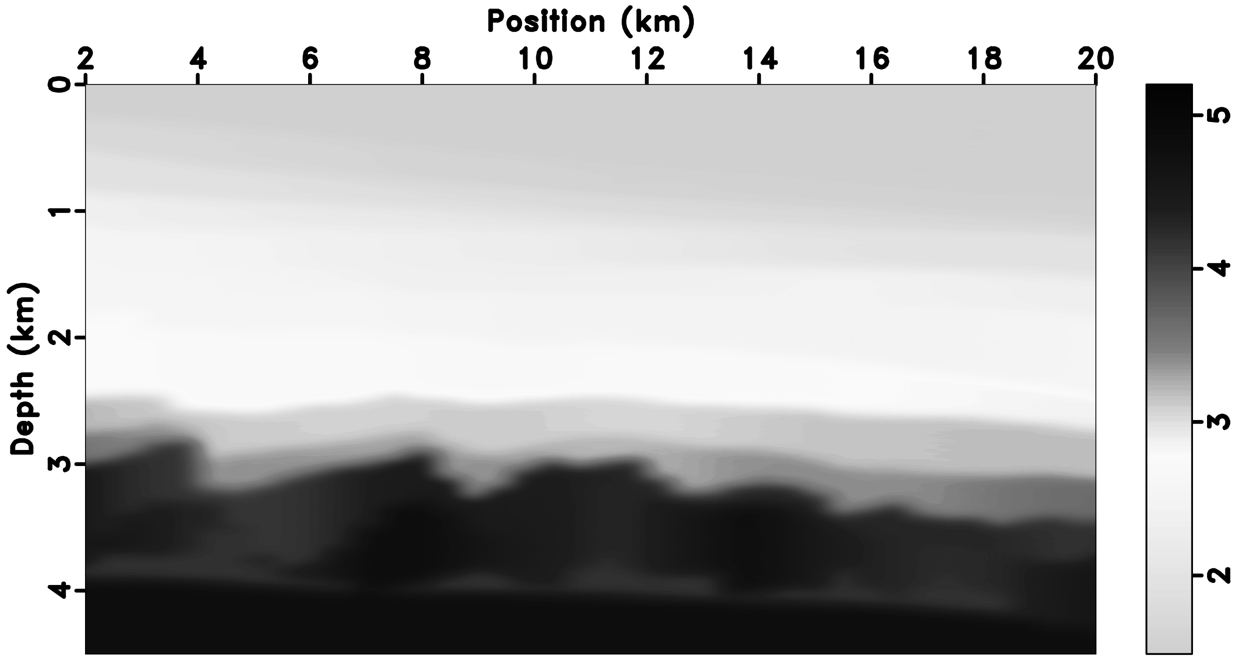

Figure 11. Velocity model for the field-data example